LEADING TODAY FOR A BETTER TOMORROW
CONTINENTAL ACADEMY

PREMIER CURRICULUM SERIES

Based on the Sunshine State Standards for Secondary Education, established by the State of Florida, Department of Education

GEOMETRY

Author: Bernice Stephens-Alleyne

Revision Date:12/2009

INSTRUCTIONS

Welcome to your Continental Academy course. As you read through the text book you will see that it is made up of the individual lessons listed in the Course Outline. Each lesson is divided into various sub-topics. As you read through the material you will see certain important sentences and phrases that are highlighted in yellow (printing black & white appears as grey highlight.) **Bold, blue** print is used to emphasize topics such as names or historical events (it appears **Bold** when printed in black and white.) Important Information in tables and charts is highlighted for emphasis. At the end of each lesson are practice questions with answers. You will progress through this course one lesson at a time, at your own pace.

First, study the lesson thoroughly. (You can print the entire text book or one lesson at a time to assist you in the study process.) Then, complete the lesson reviews printed at the end of the lesson and carefully check your answers. When you are ready, complete the 10-question lesson assignment at the ***www.ContinentalAcademy.net*** web site. (Remember, when you begin a lesson assignment, you may skip a question, but you must complete the 10 question lesson assignment in its entirety.) You will find notes online entitled "Things to Remember", in the Textbook/Supplement portal which can be printed for your convenience.

All **lesson** assignments *are* open-book. Continue working on the lessons at your own pace until you have finished all lesson assignments for this course.

When you have completed and passed all lesson assignments for this course, complete the End of Course Examination on-line. Once you pass this exam, the average of your grades for all your lesson assignments for this course will determine your final course grade.

If you need help understanding any part of the lesson, practice questions, or this procedure:

- **Click on the "Send a Message to the Guidance Department" link at the top of the right side of the home page**
- **Type your question in the field provided**
- **Then, click on the "Send" button**
- **You will receive a response within ONE BUSINESS DAY**

About the Author…

Bernice Stephens-Alleyne is a Trinidadian immigrant who is math-certified in Maryland and Florida. She started teaching school at the age of seventeen and taught within other professions as well. Ms Stephens-Alleyne received her initial teacher training in Trinidad, West Indies, at the then state-of-the-art Mausica Teachers College. Bernice continued her education at Howard University, American University, and Trinity University in Washington DC. She has taught every grade level including adults and college levels and considers her richest teaching moments at Trinity University in Washington DC. The most recent teaching experiences include teaching at-risk youth at an Alternative High School in Maryland and teaching teachers how to teach math at Trinity University. Other professional experiences have been as a business woman, an economist, union activist, and a television appearance on NBC Nightline with Ted Koppel.

Ms Stephens-Alleyne comes from a large family whose members straddle many professions. She has two adult offspring and four grandchildren at the time of this writing. Bernice enjoys reading and learning about health issues and education theory and practice. Her hobbies include nature walks, jazz music, ballroom dancing, and clothing design. Creating unusual curricula that enhance the learning process is one of her many education projects; others include math art competitions, and actively campaigning against standardized testing. Ms Stephens-Alleyne enjoys living in the Miami, Florida area.

Mathematics- Geometry
by Bernice Stephens-Alleyne

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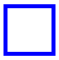
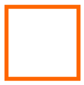
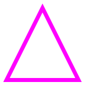
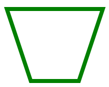
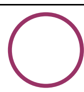
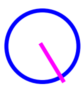
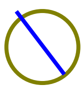
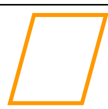
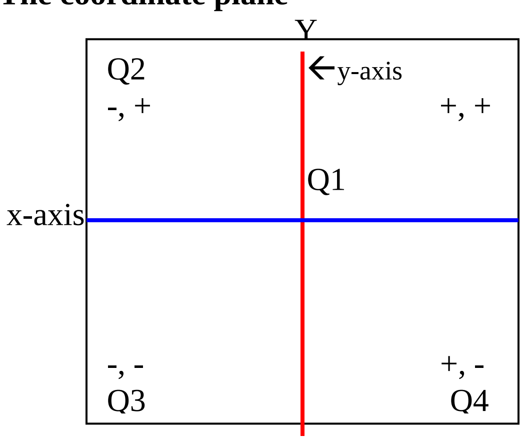
LESSON 1: HOW MUCH DO YOU KNOW?

CHECKLIST AND REVIEW

1. Measurement Formulas

Square 	Area = side x side or S^2; Perimeter = 4s
Rectangle 	Area = base x height (bh) or length x width (lw); Perimeter = 2b + 2h or 2l + 2w
Triangle 	Area = ½ (bh) or one half base x height. The height must be the perpendicular distance from the base to the tallest part. Perimeter = Sum of all sides
Trapezoid 	Area = ½ $(b_1 + b_2)$ h or one half (base one plus base 2) times height. A trapezoid has 2 parallel sides; each one will have a specific measure identified as base 1 or base 2. Perimeter = Sum of all sides
Circle 	Area = πr^2; π = 3.14 or 22/7. Use 22/7 when the radius is a multiple of 7. This eliminates the decimals. Circumference = d π or diameter times pi.
Radius 	½ d or one half diameter
Diameter 	2r or 2 times the radius; or c/π or circumference divided by pi
Parallelogram 	Area = bh or base times height; Perimeter = Sum of all sides

2. The coordinate plane



- The quadrants are numbered counterclockwise
- X values are always first
- The intersection of x-axis and y-axis is zero

3. Classification of polygons (The prefixes tell the story)

By number of sides: 3 sides = **tri**angle
4 sides = **quad**rilateral
5 sides = **penta**gon
6 sides = **hexa**gon
7 sides = **hepta**gon or **septa**gon
8 sides = **octa**gon
9 sides = **nona**gon
10 sides = **deca**gon
11 sides = **undeca**gon
12 sides = **dodeca**gon

4. Types of angles

Classification by measure of angle: 1. Acute = all angles < 90º; 2. obtuse = 1 angle > 90º; 3. right = 1 angle 90º
Classification by measure of side: 4. scalene = no equal sides; 5. isosceles = 2 sides equal; 6. equilateral = all sides equal
Combinations in a triangle: acute scalene = all angles < 90º and all sides of different lengths; acute isosceles = all angles < 90º and 2 sides equal; obtuse scalene = 1 angle > 90º and all sides of different lengths; obtuse isosceles = 1 angle > 90º and 2 sides equal; right isosceles = 1 right angle and 2 sides equal; right scalene = 1 right angle and no sides equal

5. Angle relationships

Complementary angles: Two angles with a sum of 90º
Supplementary angles: Two angles on a straight line with a sum of 180º
Equal supplements: Two right angles on a straight line, each 90º
Corresponding angles: Pairs of congruent angles formed when parallel lines are crossed by a transversal. Corresponding angles are in corresponding locations.
Vertical angles: Pairs of congruent angles opposite each other at an intersection
Alternate angles: Pairs of angles formed at alternate locations when parallel lines are crossed by a transversal
Right angles are formed at the intersection of perpendicular lines
Parallel lines are equidistant from each other and will never meet

6. Statistical terms

Mean: The simple average of a data set
Median: The middle number in a data set when the numbers are numerically arranged
Mode: The number occurring most frequently
Range: The difference between the largest and the smallest number in the data
Measures of central tendency: mean, median, and mode
Measures of dispersion: Range, variance, and standard deviation
Outlier: A piece of data found in a box and whisker plot that is at least 1.5 intervals away from the rest of the data. An outlier affects the mean of the data
Trend: The direction a graph of the data seems to take; trends are easily visible in line graphs

7. Types of graphs (9)
(a) **Line graph**: This may be straight, broken, curved, or double and may or may not be a function. Line graphs generally show some relationship between a dependent and an independent variable.
(b) **Scatter plot**: This graph is set up similar to a line graph and is used when there is no set relationship between two variables.
(c) **Bar Graph or double bar**: These compare two sets of similar data; trends are sometimes visible.
(d) **A Line plot** is set up on a number line using 'x' to identify data clusters. Each cluster will be stacked on the same value on the number line taking on the effect of a bar graph.
(e) **A Stem and leaf plot** is a strange arrangement of separating the unit value of a number from the rest of the number. Stems are written on the left of a two-column table and leaves on the right. A stem and leaf plot is a horizontal arrangement of data while the line plot is a vertical arrangement of data.
(f) **The Box and whisker plot** divides the data into quartiles (quarters) using the median as the focus point. The bulk of the data is "boxed" and the whiskers connect the lowest and the highest values. This kind of graph provides quick analysis of the way the data is spread.
(g) **A Circle graph** is a graph organized in a circle using degrees to divide the data. Small data sets are best analyzed using this method.
(h) **A Histogram** looks a lot like a bar graph and does similar work. There are no spaces between each value in a histogram and in that sense the graph demonstrates continuous "history" of the data.
(i) **A Pictograph** (picture graph) is one of the first graphs students learn. Most kindergarteners create pictographs. As the name implies, pictures are used to represent clusters of data which is often represented horizontally.

8. Equivalent number formats
Numbers can be expressed in many different formats: Whole numbers, mixed numbers, decimals, percents, fractions, ratios, scientific notation, standard notation, squares, square roots, and exponents.

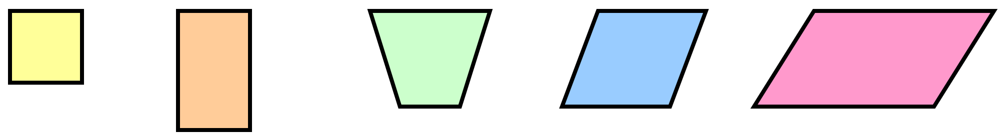
9. Managing information
We use tables, graphs, charts, and matrices to organize and manage information and make analyses and comparisons. Each method has its distinct advantages and disadvantages because it serves a particular purpose. No one method is perfect but some are used more prevalently than others because they provide advantages in retrieval compared to the others. Errors can occur with each type.

10. Use this review as a reference guide. Many of these basics are fully developed in the Pre-Algebra and Algebra 1 curricula of this series.

LESSON 1-A
INDUCTIVE AND DEDUCTIVE REASONING
Differences and examples

Both types of reasoning will be used throughout the text. Inductive reasoning takes an **investigative** approach using a problem or a situation to "reveal" the theory embedded in the activity.

Example: Look at the pictures and find words to group them.



After many trials we choose **4-sided** because they all have 4 sides. One word for all 4-sided figures is quadrilateral. We have **observed some pattern**, come to a **conclusion**, and made a **conjecture**. This is the essence of inductive reasoning. Geometry is full of the inductive reasoning approach.

What is inductive reasoning? We have used inductive reasoning to find the definition. Look at the words in red to make a statement about inductive reasoning. Inductive reasoning is when you _____________, ______________, come to a ____________ and make a ________________.
You should have these words in the blank spaces: investigate, observe a pattern, conclusion, conjecture.

Deductive reasoning is the opposite. It states the **theory** or gives the **conjecture**, then provides **examples** to illustrate. Instead of showing pictures first in the example above, deductive reasoning would state that all 4-sided figures are quadrilaterals, and then show the pictures.

What is deductive reasoning? Deductive reasoning provides the _______________ then gives ___________________ to explain.
You should have these words in the blank spaces: theory, examples.

Book Organization

The book will have a mix of inductive and deductive reasoning to ensure that each student has the opportunity to learn from both techniques. Examples will be in **bold type.**

New or unusual vocabulary and key words or phrases will be highlighted. If the curriculum is printed in gray scale the highlights will still be clear. There will be full or partial solutions to examples or practice problems. This will occur if the examples are used to reinforce an explanation or if the problem imitates an example fully explained.

Interactive web sites will enhance the learning process.

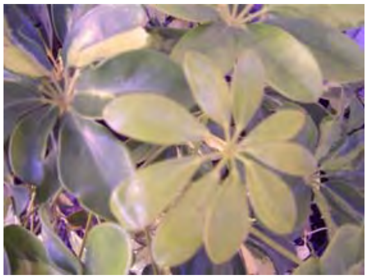
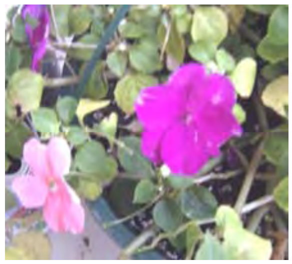
LESSON 2
THE WIDE WORLD OF GEOMETRY

The word geometry can be broken into its prefix, ***geo*** and its root, ***metr-***. Geo means earth and metr- or meter means measure. Geometry means earth measure. It literally means that. Everything on earth can be measured using geometry. Everything we measure comes in patterns; so much of geometry is about finding the pattern.

Geometry in nature

(All pictures taken by the author)

Fibonacci number of leaves or petals in these three


Symmetry in the leaf arrangement

The pentagonal 5 of the starfish

Geometry in Auto Engineering and Street Signs

Look for the use of geometric figures in the logos found on automobiles. The circle, pentagon and oval are just a few figures used. Street signs and other directional signs also make use of such geometric figures as squares, rectangles, circles, triangles and octagons. Can you identify these in the real world?

…And Architecture

Architecture makes great use of geometric figures. Look foe examples such as parallel lines, triangles, rectangles, squares, arcs and others in various structures in and around your neighborhood.

Geometry in Nature: The Fibonacci number pattern is evident in nature. What is the Fibonacci number pattern? We'll do it deductively at first!
1, 1, 2, 3, 5, 8, 13, 21, 34, 55…
(a) Now we will take an inductive look.
(b) How do we get the next number in the series? Is there a consistent pattern?

Answer to (a) Add the last two numbers to get the next one.
(b) There is no consistent pattern.

The Fibonacci numbers are found in arrangements of petals, the starfish, pinecone spirals, pineapple spirals, fractal fruits and vegetables (broccoli and cauliflower), and so on. The number 5 is quite common. So is the number 8 (see the green and white leaves). 13 is also evident often in nature (black-eyed susan flower). Roses often have a Fibonacci number of petals at maturity. Explore the web site, http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fib.html (Accessed: 1/23/2006), for more on this series.

Geometry in Art: The tangram is one of the old art forms that taught math; it is still used today in many classrooms. It is one of my favorite teaching tools to demonstrate fractional concepts. There are **square, rectangular, and circular tangrams**. Magic squares are depicted as art in many museums and in the books of math artists and historians. Tessellations, which will be explored in some depth later in the curriculum, are fascinating tiled drawings and constructions using translations, rotations, reflections, and symmetry. Kitchen floor tiled designs are good examples of tessellations. Patterns and sequences form the basis of many musical instruments such as the harp, piano, guitar, and others; geometry is also embedded in the theory of sound, range of notes, and musical scales. Click on the websites to discover more of the exciting world of geometry in art.

http://www.georgehart.com/sculpture/sculpture.html (Accessed: 1/23/2006)
http://www.root2art.co.uk/word/geometric_art.html (Accessed: 1/23/2006)

Mini Project

Geometry Hunt

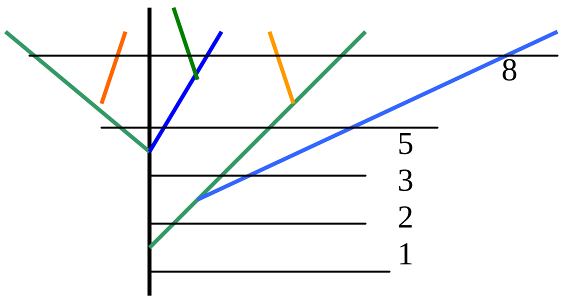
Find 20 objects that fit these categories:
3 must be from nature and have different Fibonacci numbers
5 must be from around the house and have different geometric shapes
4 must be automobile logos that have combination shapes.
5 must be from building architecture
3 must be from street signs: 1 octagonal, 1square or rhomboid, and 1 rectangular

This is a fun project. Be creative. Collect your items, arrange them artistically, and explore the **WIDE WORLD OF GEOMETRY**.

LESSON 2-A
FIBONACCI, PASCAL, AND MAGIC SQUARES

We experienced the **Fibonacci sequence** in the section on geometry in nature. Let's explore in a little more detail.

The 1, 1, 2, 3, 5, 8… comes from the way young plants grow in a spiral of new leaves. The stick diagram counts the new growth at the cross section.



Many flowers have 5 petals; others have 13 or even 21. Vegetables like broccoli and cauliflower have fractal growth, a constant breaking and branching out into smaller units. Those examples have this fascinating number arrangement as well. Do your own exploration and discover the properties of this number sequence. On your own time research the name Fibonacci and learn how this number sequence was popularized.

Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

Get the picture, OOPS, pattern?

Why is this important in geometry? Deep in this arrangement of numbers is the binary number system, base 2, FOIL, the counting numbers, triangular numbers, the Fibonacci number sequence and more.

Inductive Activity

1. Number each row starting with row zero. There should be 7 rows with this system of numbering.
2. Count the number of numbers in each row.
3. Add the numbers in each row.
4. Look at the second number of each row.
5. Look at the numbers at the diagonal on each edge.
6. Study the numbers on the second diagonal on each side.
7. Find a row of triangular numbers.
8. Study row 2. How did the '2' get there?
9. Can you complete the next 3 rows?

Observations, Conclusions, and Conjectures

Numbers in each row			Sum of the numbers in each row	
Row #	Number of numbers		Row #	Sum of the numbers
0	1		0	1
1	2		1	2
2	3		2	4
4	5		3	8
5	6		4	16
Conclusions: Add 1 to the number of the row to find out the number of numbers in the row.			Double the last row to get the sum of the numbers in the next.	

We can write the numbers in each row as a formula. If ***n*** is equal to the number of the row, what is the formula for the number of numbers in the row?

If ***n*** is equal to the number of the row, what is the formula for finding the sum of the numbers in each row?

Magic Squares on a 3 x 3 grid

8	**1**	**6**
3	**5**	**7**
4	**9**	**2**

The sum of each column, row, or diagonal should be the same. That number is the **magic number**

The key is where to place each number because they are arranged in a spiral pattern. We will do this one deductively by providing the solution first. Then you can make one of your own. Note that you can make 4 x 4 grids, or 5 x 5 grids. These, of course are more difficult.

Make your own Magic Square

The sum of each column, row, or diagonal should be the same. That number is the **magic number**

1. Create a series of 9 consecutive numbers. You may choose odds, evens, multiples of 3 or 5, or a mix of negative and positive numbers. Any series will work as long as it is an arithmetic sequence.
2. Place each number according to the order in the magic square above.
3. Add each row, column, or diagonal. Did you get the same answer? That is your magic number.

Magic Square Activity

1. Place each number in the appropriate box and find the magic square: 3, 6, 9, 12, 15, 18, 21, 24, 27
2. Find the missing numbers in this magic square and write the magic number.

	0	**7.5**
	6	
4.5		**1.5**

3. The magic number is **-12.** Complete the blanks.

		-6
0		
		2

Solution to Pascal's Triangle and Magic Squares

Inductive activity:

- Answers to #2 and #3 are on page 10.
- The second number of each row is a counting number. The counting numbers are arranged consecutively. The second number of each row is the number of the row.
- The numbers on the diagonal are all ones.
- Same as number 4.
- The 3rd diagonal is the triangular number sequence.
- Add the two numbers above to get the one in the row below. The sum of 1 and 1 gave the 2 below. In row 3, the sum of 1 and 2 gave the 3 below it. Follow the color coding.
- The next 3 rows: 1, 8, 28, 56, 70, 56, 28, 8, 1 **(row 8);** 1, 9, 36, 84, 126, 126, 84, 36, 9, 1, **(row 9);** 1, 10, 45, 120, 210, 252, 210, 120, 45, 10, 1 **(row 10)**
- Formula for the number of numbers in each row = **$n + 1$**
- Formula for the sum of the numbers in each row **= 2^n where n is the number of the row. The second row would be 2^2, row 3 would be 2^3.**

Magic Square Activity

24	3	18
9	15	21
12	27	6

The magic number is 45

10.5	0	7.5
3	6	9
4.5	12	1.5

The missing numbers are in red. The magic number is: 18

-10	4	-6
0	-4	-8
-2	-12	2

The magic number is -12. The missing numbers are in red.

On Your Own

1. Spend some time researching the origin of Magic Squares. You will be amazed at the exciting Math History contained in this aspect of Geometry.
2. Who was Fibonacci and how did he discover these numbers?
3. What was Pascal's complete name and where was he born? Where is the FOIL in Pascal's Triangle?
4. Did you know that Pascal's Triangle and Fibonacci are part of the 7th grade curriculum in many school districts?

LESSON 3
GEOMETRIC SKETCHES AND CONSTRUCTIONS
LESSON 3-A

Geometric Drawing Tools

Here are a few different drawing and measuring instruments used in geometry:

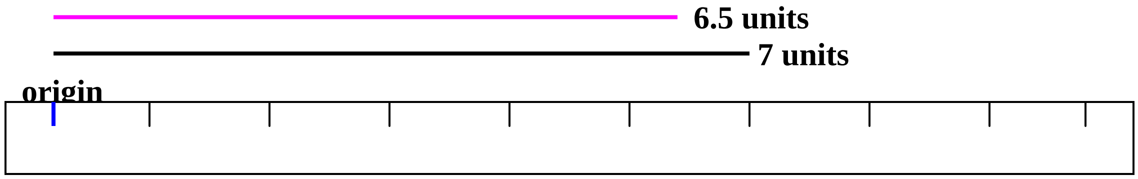
- The ruler or straight edge is used for drawing straight lines or line segments. Any object with a straight edge will suffice.
- The protractor which may be half a circle, a complete circle, or contained in a ruler, is used for measuring degrees of an angle. These optional web sites demonstrate measuring degrees of an angle with a protractor.

  http://www.myjanee.com/protractor.htm (Accessed: 1/23/2006)
  http://www.eece.ksu.edu/~hkn/files/protractor.pdf (Accessed: 1/23/2006)
  http://www.ehow.com/how_12928_protractor.html (Accessed: 1/23/2006)

- The compass is used to construct circles and arcs (fractions of the circumference). This website demonstrates many of the tools used in geometry and the compass is among them. There may be plastic, metal, or wooden compasses. One has to be aware of possible injury to oneself if the compass has a metallic point.
- The T-Square once the most widely used of all geometric instruments is now used mainly by architects. It is great for measuring straight lines, parallel lines, and angles that are close together.
- The Vernier caliper, an old instrument, can measure the depth of an object when your hand is unable to reach inside.
- When you buy a complete geometry set you get two or three triangles, a 6-inch ruler, a compass, a small pencil, and so on. The triangles were meant to be used in different ways. Sometimes a pair of them placed in a particular way can create a perfect parallelogram; one can draw a right triangle or an isosceles triangle. These triangles are called set squares.

LESSON 3-B
Creating Sketches and Constructions

1. **Drawing lines and line segments**: Rulers may have metric and standard measurements. The trick is to ensure that you start at the **origin** which may ***not*** be the edge of the ruler. The origin is the **first point of measurement** and is actually zero.



The metric measures are closer together than the standard measures. 10 little marks on the ruler measure 10 mm or millimeters or 1 centimeter (cm). Often as a teacher when I asked my students to use centimeters, I would get responses like, "I have only millimeters on my ruler." If you have millimeters, you have centimeters. The metric side can give measures using decimals as well. Each small space is equal to **.1 of a centimeter**
The standard measures are inches and may be divided into 4 parts. Each fourth is ¼ of an inch. One can get measures like 2 ½ inches or 8 ¾ inches. Always pay attention to the side of the ruler you use to ensure accuracy in measurement.

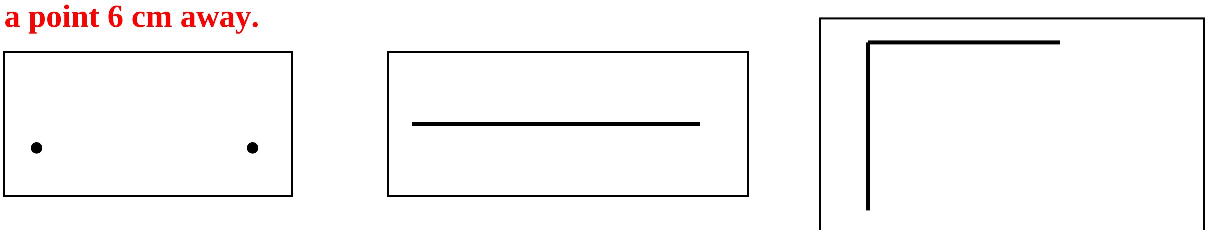
2. **Drawing versus sketching**: Both these terms will be used and there is a difference. Sketching is drawing without tools or freehand. Often a sketch is needed to "see" what the final drawing should look like. Sketched lines and angles may not have accurate measurements.

3. **Drawing versus construction**: Drawing is done using a ruler or straight edge. Construction is done using a protractor, compass, vernier caliper, T square, or other instruments.

4. **Constructing planes**: One can construct planes using a ruler, protractor, compass, set squares, or T squares. Let's try a rectangular plane and a triangular plane.

(a) A rectangular plane must have 2 pairs of equal sides and these sides must be parallel. Pick specific measurements; say 6cm by 8 cm. We will construct using ruler, pencil, and the right triangle.

Step 1: Use the ruler to find the point of origin; place that point on the paper. Mark a point 6 cm away.



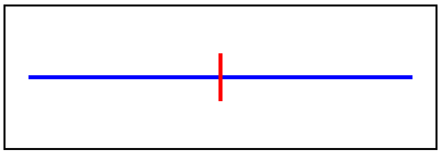
Step 2: Join the dots

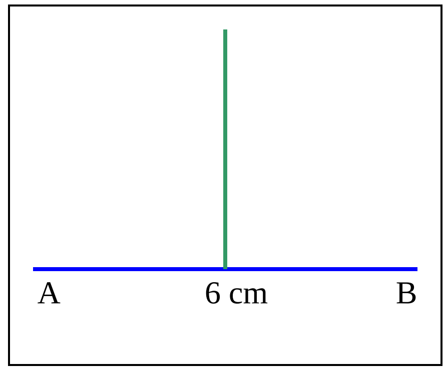
Step 3: **Use the right triangle at the corner and draw vertically; use the ruler to measure a point 8 cm away from the corner; draw towards that point**
Step 4: Use the ruler to draw the 6 cm side.
Step 5: Connect the ends for the second 8 cm side.

(b) Constructing a triangular plane: One can use a ruler, compass, or any of the triangles in the geometry set. If the triangle has specific measurements, then the job becomes more meticulous. Let's say we are constructing an isosceles triangle with a 6cm base and sides 4 cm sides.

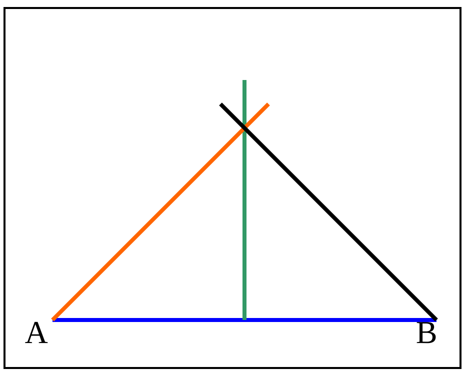
Follow steps 1 and 2 above.

Step 3: Use the ruler to find the midpoint of that line and place a point or marker at 3cm.





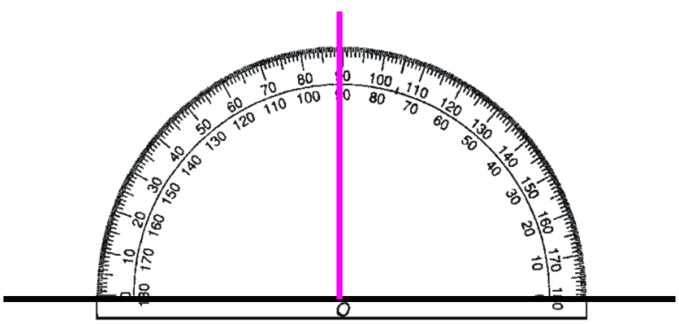
Step 4: Draw a soft line from the mid-point You will have to erase this line later.

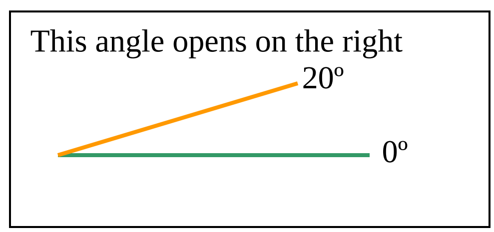


Step 5: Use the ruler to construct a line 4 cm from point A to a point on the vertical line. Repeat the steps for point B

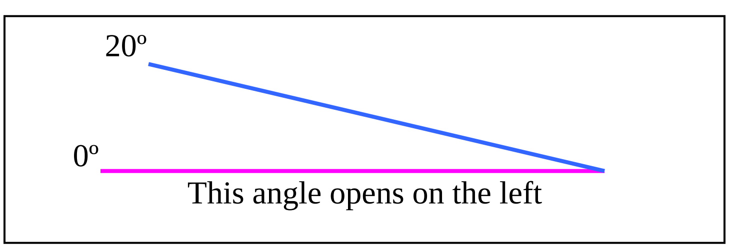
Step 6: Erase all overlapping marks and the mid-line.

5. **Constructing angles**: We use a protractor to construct angles.

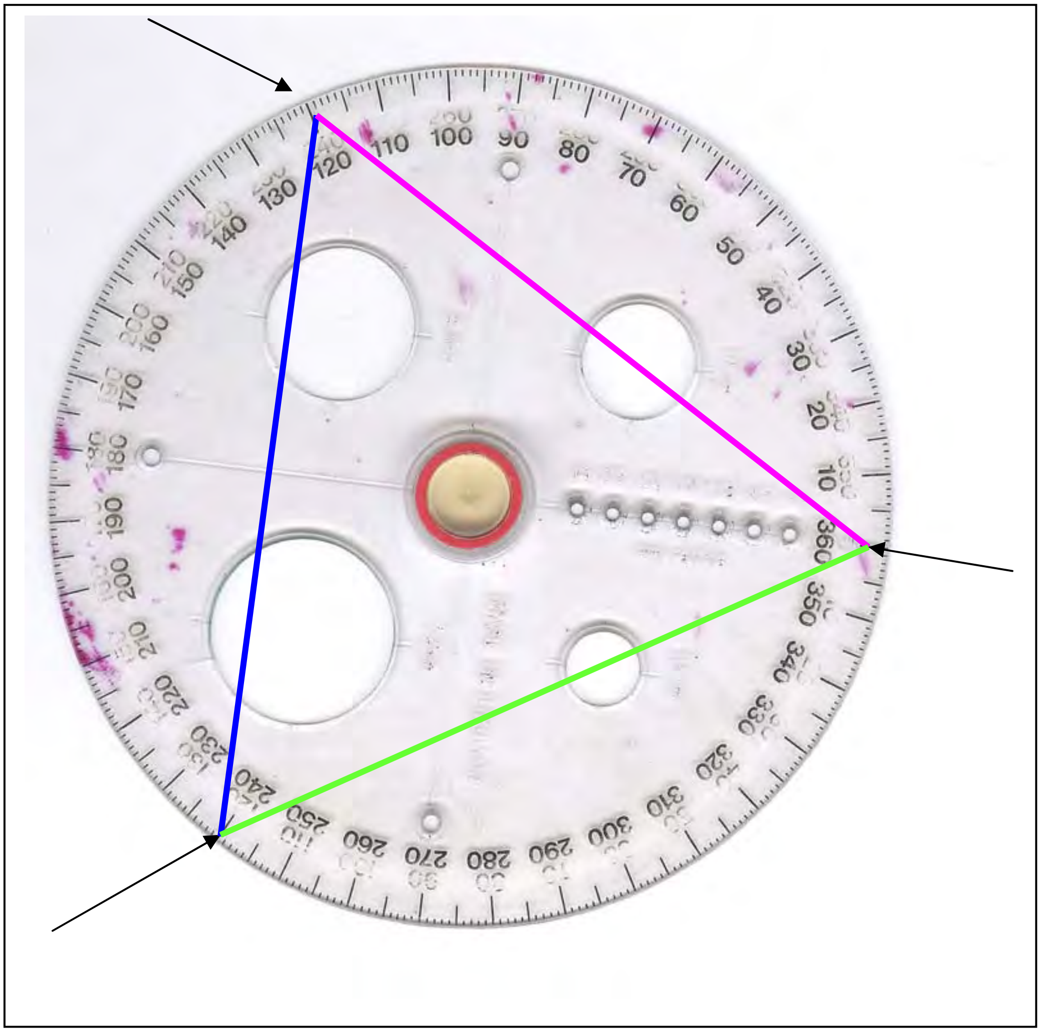




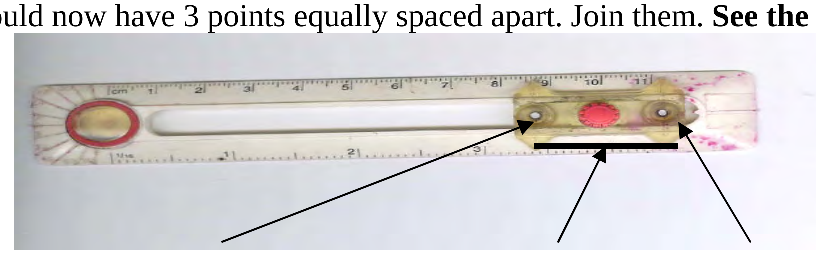
The black line represents the zero line. One ray of the angle must be on this line. The vertex of your angle must be at the center of the protractor. The trick is in counting the degree measures. **If the angle opens on the right, start counting from the right. If the angle opens on the left start counting from the left. It is OK to turn the angle upside down if that helps.**



6**. Constructing polygons and non-polygons:**
(a) Equilateral triangle: We will explore 2 ways. One way is to use a circular protractor and mark points 120º, 240º, and 360º. We choose these points because they are exactly the same distance apart. The same distance apart in a circle means that the arcs (parts of the circumference) are the same degree measure. If the arcs are the same measure, the angles opposite the arcs are the same measure. The word ***"equilateral"*** means equal measure.

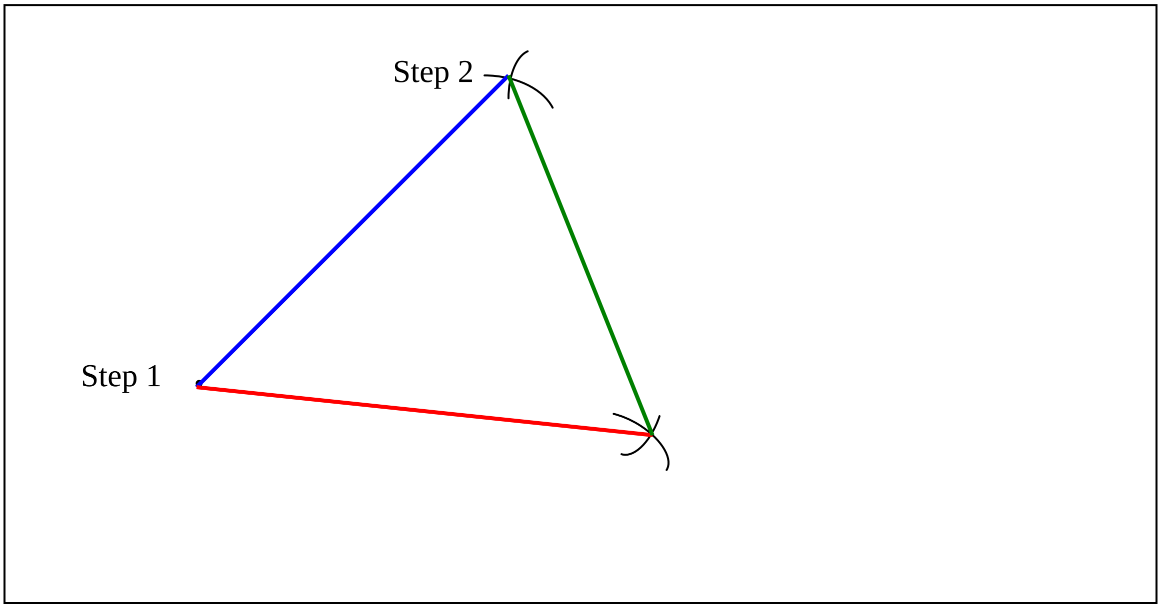


Another way would be to use a compass. Mark a point on your paper; with that point as center use any radius to make an arc. Place a point anywhere on that arc. With that point as center, and the same radius as before, make another arc. Go back to the first point, keep the same radius and make an arc that crosses one of the arcs you already have. You should now have 3 points equally spaced apart. Join them. **See the picture below**.

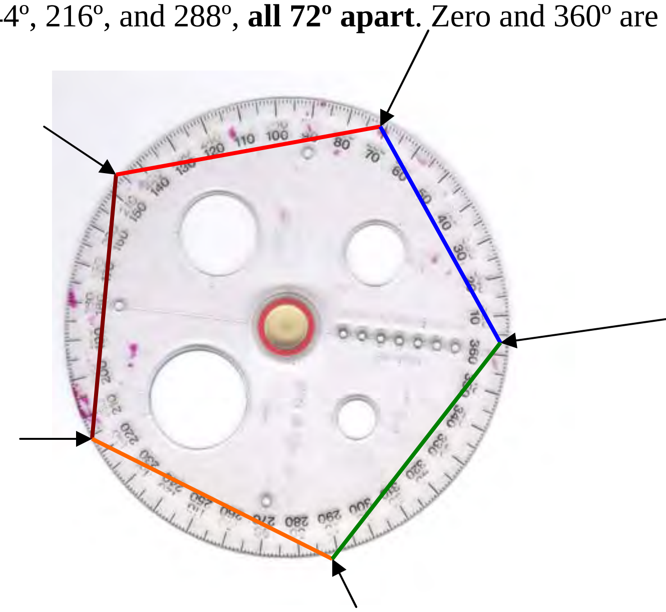


This compass is contained inside a ruler.

Center of compass 2-inch radius pencil goes here

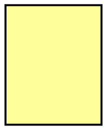


(b) Regular pentagon: Use the circular protractor which contains 360º. Divide 360 by 5; that would be the measure of each arc (72º). Start at zero, place a point; the next points would be at 72º, 144º, 216º, and 288º, **all 72º apart**. Zero and 360º are on the same point.

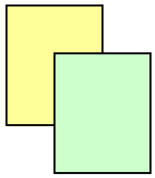


(c) Circles and circular objects are non-polygons. Any of these may be constructed with a semi-circular protractor or a full circle protractor. Simply outline around the shape to the desired length. Be aware that semi-circles must start at zero and end at 180º. An entire circle can be drawn with 2 semi-circles. Certain artistic pieces like the daisy design and the mandala can be constructed using a compass or a semi-circular protractor. Explore websites for doing these at your leisure.
http://www.artlandia.com/wonderland/ (Accessed: 1/23/2006)
http://www.abgoodwin.com/mandala/links/creating.html (Accessed: 1/23/2006)

7. **Constructing 3-D objects**: Three-dimensional objects have perspective. You can see only parts of them and allow the imagination to visualize what is not seen. Let's try the rectangular prism (box) and the square pyramid.

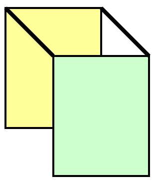


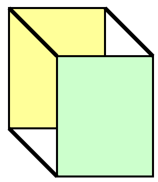
Start with a rectangle

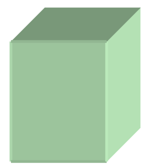


Draw another rectangle in front of it.

Connect the corners at the top and bottom. Erase all the lines that make it look transparent.

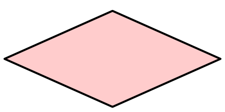


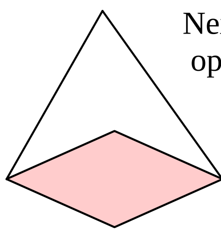




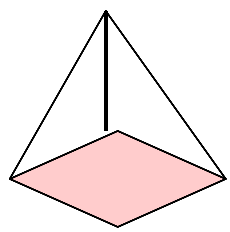
Computer version

Start with a square that is stretched out.



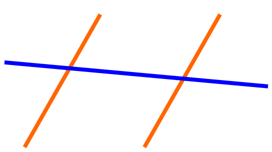


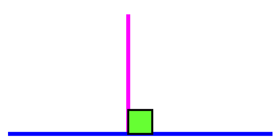
Next, draw a 2 sides of a triangle on the opposite corners.



Draw the 3rd side of the triangle and there is your pyramid!
We cannot see the 4th side.

8**. Constructing parallel line relationships:** A ruler is good for one pair of parallel lines if you draw on either side of it. Draw another line crossing it and we have a transversal. This arrangement will provide pairs of congruent, corresponding angles, interior alternate and exterior angles, vertical angles, and supplementary angles.





9**. Constructing perpendicular line relationships:** A right triangle would be the tool to use. When a line is perpendicular to another a right angle is formed. The height of a triangle is perpendicular to the base. Supplementary angles may also be formed but since they are both 90º, they are called equal supplements. A small box at the vertex of a right triangle indicates the degree measure.

10. **Constructing circles and circle parts**:

C is the center of the circle.

Circumference is a line around the circle (all points on the circumference are equidistant from the center point.)

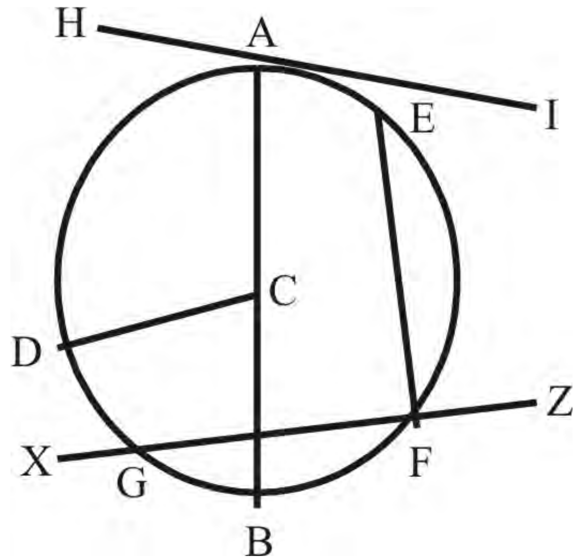
Radius (r) is a line from the center to the Circumference of the distance from the center to the circumference. CD is one such radius (r).

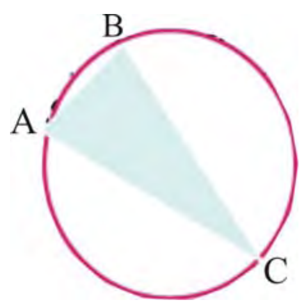
Diameter (d) is a straight line from one point on the Circumference through the center to another point on the circumference. AB is one such diameter (d).

Chord is a line that touches the circumference in two places. Lines GF, EF and AB are chords. Note that AB is also a diameter.

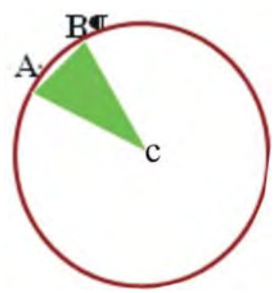
Tangent is a line that touches the circle in exactly one point. HI is a tangent at point A.

Secant is like a chord except that it runs through the circle like line XZ.

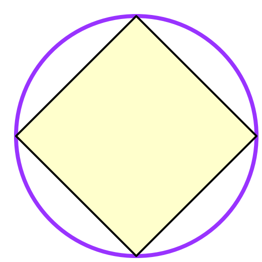




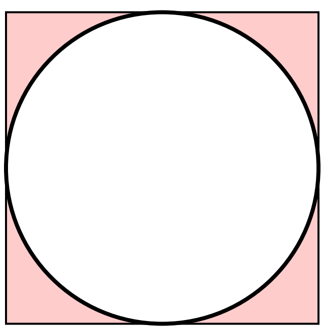
Inscribed angle ACB comes from inside the ***circumference***



Central angle ACB comes from the ***center***



Circumscribed circle or circle drawn ***around*** a square



Inscribed circle or circle drawn ***inside*** a square

LESSON 3
REVIEW OF LESSONS 1, 2, AND 3

1. How is the formula for finding area of a triangle similar to that of the area of a trapezoid?
2. When is it appropriate to use $^{22}/_{7}$ for pi?
3. How are circumference and perimeter similar; how are they different?
4. Where would the coordinates (-2, 3) be found: (a) Quadrant 1 (b) Quadrant 2 (c) Quadreant3 (d) Quadrant 4?
5. What is the value of the intersection of x and y?
6. Polygons are classified by the number of sides. What part of their name tells the number of sides?
7. A triangle had 2 equal sides and 1 right angle. Name it.
8. It is possible to have a scalene right triangle. T/F
9. It is possible to have an obtuse right triangle. T/F
10. Name one combination of triangle that is not possible.
11. What 4 types of angles are formed at the intersection of transversals and parallel lines?
12. Complementary angles equal 180º. T/F
13. What is an outlier and in which type of graph would it be found?
14. The data in a graph seemed to be increasing. That phenomenon would be called (a) slope (b) trend (c) mode (d) distance
15. Which graph divides the data into quartiles?
16. Which type of graph is good for small data sets and uses degrees to separate the data?
17. Name 6 equivalent number formats.
18. A project took an investigative approach and provided examples to observe. This was inductive/deductive reasoning. Circle the correct choice.
19. One makes a conjecture at the end of a deductive reasoning activity. T/F
20. Complete: Key words or phrases will be ______________________
21. Why are interactive websites important?
22. Many plants have 5 petals. This is a common ___________________ number.
23. Write the next 3 numbers in the series: 5, 8, 13, 21, 34, _____________, ________, ___________.
24. Name 3 aspects of nature where a Fibonacci number can be found.
25. Tessellations can have (a) translations (b) dilations (c) reflections (d) all of the above
26. How many numbers would there be in row 8 of Pascal's Triangle?
27. What formula provides the answer for the number of numbers in any row of Pascal's Triangle?
28. With an arrangement of Pascal's Triangle one would find (a) squares and triples (b) triangular numbers and square numbers (c) counting numbers and triangular numbers (d) None of the above
29. The sum of the numbers in row 6 is 2^6. T/F
30. 2^n can be used to find the sum of the numbers in any row of Pascal's Triangle. T/F
31. The middle number in row 8 is 56. T/F

32. If the sum of the numbers in row 5 is 32, then the sum of the numbers in row 6 is 3 times 32. T/F
33. I can use a series of any 9 numbers for the magic square. T/F
34. Fibonacci numbers will not work in a Magic square because ________________
35. Name 3 different drawing tools used in geometry.
36. Geometry means measure the sea. T/F
37. An instrument used to measure angles is a ___________________
38. A circular compass can (a) measure angles (b) construct circles (c) both a and b (d) neither a nor b.
39. An arc is part of the _______________ of a circle.
40. A ruler or compass can be used for (a) drawing (b) sketching (c) Both a and b (d) Neither a nor b.
41. Drawing and constructing can sometimes mean the same thing. T/F
42. If an angle opens to the right, start counting from the left. T/F
43. Use a line and its mid-point to create a right triangle. T/F
44. A circular compass can be used to construct regular polygons. T/F
45. Complete: The height of a triangular is _________________________ to the base.
46. Complete: The central angle is formed at the _________________ of a circle.
47. Complete: An inscribed angle is formed inside the circle at the ______________.
48. What is the difference between a tangent to a circle and a secant?
49. The word circumscribed means _________________________
50. Complete: A circle drawn inside a square touching it once on each side is _______________ in the square.

Solutions and Partial Solutions

Unanswered questions should be researched in the appropriate unit

2. Use 22/7 when the measure of the radius or diameter is a multiple of 7.
6. The prefix tells the name of the polygon.
7. Isosceles right triangle. 8) T 9) F 10) Acute right
11. Vertical, alternate, corresponding, supplementary
13. An outlier is a piece of data that is at least 1.5 intervals away from the rest of the data. It would be found in a box and whisker plot.
14. (b) trend 15) box and whisker plot 16) circle graph
17. Fractions, scientific notation, standard notation, decimals, exponents, whole numbers, percents, mixed numbers, squares and square roots.
18. Inductive 19) F 22) Fibonacci 23) 144 26) 9 numbers 27) n + 1
28. (c) counting numbers and triangular numbers 29) T 30) T 31) F
32. F 33) F 34) Fibonacci numbers will not work in a Magic Square because they are not an arithmetic sequence 38) (c) 39) Circumference 42) F 43) F
44. T 45) Perpendicular 46) Center 47) Circumference
48. The difference between a tangent to a circle and a secant is that the tangent sits outside the circle touching it at exactly one point but the secant goes through the circle crossing the circumference two times.
49. The word circumscribed means drawing around.
50. Inscribed in the square.

LESSON 4
ALL ABOUT POLYGONS
LESSON 4-A

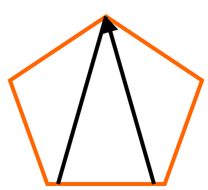
Classification of Polygons: Refer to the checklist and review on page 4.

Properties of regular polygons: All sides and angles are congruent in regular polygons.

Name of polygon	Number of sides	Measure of each angle	Sum of the interior angles	Formula for finding sum of the interior angles
Equilateral Triangle	3	60º	180º	(n-2)180º
Square	4	90º	360º	(n-2)180º
Regular Pentagon	5	108º	540º	(n-2)180º
Regular Hexagon	6	120º	720º	(n-2)180º
Regular Octagon	8	135º	1080º	(n-2)180º

The formula for finding the sum of the interior angles of a regular polygon is (n-2)180, where ***n*** is the ***n***umber of sides.
Example 1: An equilateral triangle has 3 sides; therefore n = 3; n -2 = 3-2 = 1; 1 · 180 = 180º
Example 2: A regular hexagon has 6 sides; n = 6; n – 2 = 6 – 2 = 4; 4 · 180 = 720º

How did we get this formula? The old masters found that one could divide each regular polygon into a number of triangles and multiply the number of triangles by the measure of degrees in 1 triangle. Let's see what that looks like:



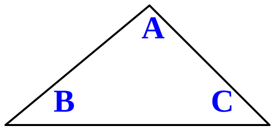
Draw diagonals from 1 point to each vertex to determine the number of triangles in a pentagon. We get 3.
n – 2 = 5 – 2 = 3. 3 x 180º = 540º

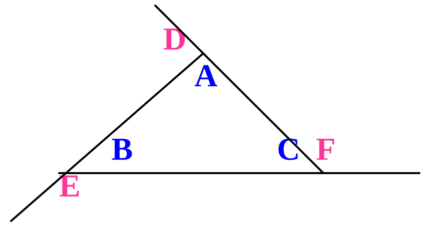
Example 3: How many triangles are there in a 12-sided polygon and what would be the sum of the interior angles of a dodecagon? What is the measure of each angle of a regular dodecagon?
Apply the formula (n-2)180º = (12 – 2)180º = 1800º
The sum of 10 triangles (1800) ÷ 12 angles = 150º

Knowledge of the sum of the interior angles of a polygon will assist us in other ways:

- The measure of each interior angle of a regular polygon
- The measure of each exterior angle of a regular polygon

Interior and exterior angles: Interior means inside and exterior means outside. Interior angles are the angles inside a polygon at each vertex; exterior angles are the angles formed outside a polygon.





What type of angles that we have already studied is formed at D, E, and F? (**Think angles on a line**)

Theory (Deductive approach): The sum of the exterior angles of any polygon, regular or irregular, equal 360°.

1. Find the sum of the exterior angles of a square. ________________
2. Find the measure of each exterior angle of a square.______________
3. What name would you give one pair of exterior and interior angles formed at the same vertex, such as A and D, or E and B?

Solutions: 1) 360° 2) 90° 3) Supplementary

In a regular polygon interior angles would be congruent to each other!
In a regular polygon exterior angles would be congruent to each other!

Inductive Activity

Observe the pattern, complete the blanks, and write conjectures as appropriate

Regular polygon	Sum of interior angles	Measure of each interior angle	Sum of exterior angles	Measure of each exterior angle
Equilateral triangle	180°	60°	360°	120°
Square	360°	90°	360°	90°
Regular pentagon			360°	
Regular hexagon		120°		
Regular decagon	1440°			
Regular 15-sided polygon		156°		24°

Make conjectures about each of the following:
1. Exterior angles of a polygon
2. The variation in exterior angle size as the polygon gets larger
3. A formula for finding the measure of an exterior angle of a regular polygon
4. The relationship between the sum of the exterior angles of a polygon and a circle

How to construct exterior angles: Exterior angles are **supplementary to the interior angle** on the same line. Imagine that each side of a polygon is extended beyond the interior angle creating a pair of supplementary angles.

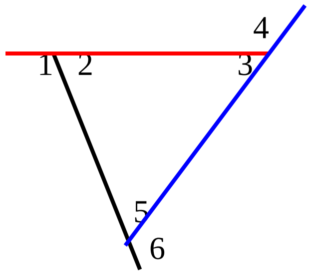


Diagram A
Angle 1 is supplementary to 2, 3 is supplementary to 4, and 5 is supplementary to 6. If the measure of an interior angle is known, then it is easy to find the measure of the exterior angle that is supplementary to it.

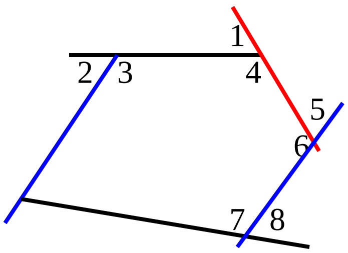


Diagram B shows exterior angles of an irregular pentagon.

Practice drawing the exterior angles of a rectangle, square, and regular or irregular hexagon. Measure the interior and exterior angles. Add the interior angles; add the exterior angles. What do you observe about the results? How do they compare with the data in the table on page 27?

Calculate angle measures

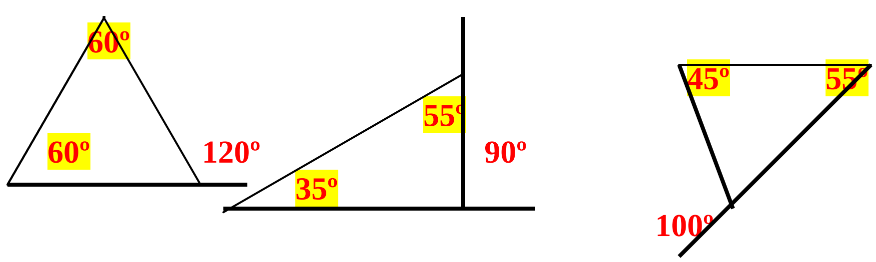
Use diagram A

1. If angle 1 = 110º angle 2 = __________________
2. Angle 3 = 60º; therefore Angle 4 = _______________
3. What is the measure of angle 5? __________________
4. Find the measure of angle 6 _____________________

Use diagram B

5. Two angles are not numbered. Name the interior angle 9 and the exterior angle 10. Find their measures at the end of this activity.
6. Angle 1 = 50º; angle 3 = 100º; angle 8 = 60º; angle 6 = 95º. Find the measures of angles 2, 4, 5, and 7.

Remote interior angles of a triangle



Add the **two remote interior** angles. Compare the result with the measure of the exterior angles. Complete: The exterior angle is equal to ___________________________________

Solutions to questions on pages 27 and 28

Regular polygon	Sum of interior angles	Measure of each interior angle	Sum of exterior angles	Measure of each exterior angle
Equilateral triangle	180º	60º	360º	120º
Square	360º	90º	360º	90º
Regular pentagon	**540**	**108**	360º	**72**
Regular hexagon	**720**	120º	**360**	**60**
Regular decagon	1440º	**144**	**360**	**36**
Regular 15-sided polygon	**2340**	156º	**360**	24º

Conjectures

1. The sum of the exterior angles of a polygon equal 360.
2. As the polygon gets larger the measure of the exterior angles gets smaller
3. Formula for finding the measure of an exterior angle of a **regular polygon**: 360 ÷ n, where n = number of angles in the polygon.
4. The sum of the exterior angles of any polygon = the degree measures of a circle.

Calculate angle measures

Diagram A: 1) 70 2) 120 3) 50 4) 130
Diagram B: 5) Angle 9 = 95; angle 10 = 85 6) angle 2 = 80; angle 4 = 130
Angle 5 = 85; angle 7 = 120

Remote interior angles

The exterior angle of a triangle is equal to **the sum of the two remote interior angles**.

Take another look at the angle measures in the diagram at the bottom of page 28; compare the measure of the remote exterior with the two remote interior angles. Is the statement above true?

Web Sites to practice or get more help

http://www.sparknotes.com/math/geometry2/theorems/terms/term_14.html
http://regentsprep.org/Regents/math/math-topic.cfm?TopicCode=poly
http://www.mathleague.com/help/geometry/angles.htm
http://www.math-magic.com/geometry/int_ext_angles.htm
(All websites Accessed: 1/23/2006)

LESSON 4-B

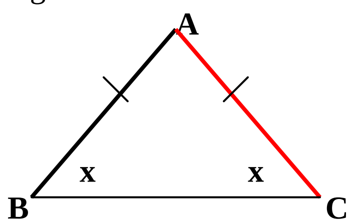
Review of triangle types: Refer to page 4 of the review section.

Construction Activity

Use appropriate geometric tools to construct the following triangles:

1. Right scalene 2. Obtuse isosceles 3. Acute scalene
4. Obtuse scalene 5. Isosceles right 6. Acute isosceles
7. Equilateral

Previously, you may have discovered that angles opposite congruent sides are also congruent. Let's see if that works with the isosceles triangle.



ABC is an isosceles triangle with AB AC. Angle B is opposite side AC and angle C is opposite side AB. Angle B = Angle C.

In an isosceles triangle (2 sides equal) the base angles are congruent to each other. This is the isosceles triangle theorem

Construction Activity (Inductive)

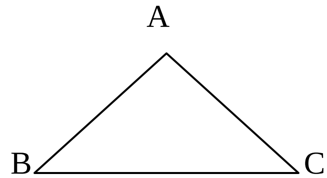
Construct 4 triangles:
Triangle 1: 4cm, 5 cm, and 6 cm
Triangle 2: 3 cm, 5 cm, and 10 cm
Triangle 3: 5 cm, 8 cm, and 10 cm
Triangle 4: 5 cm, 7 cm, and 9cm

State your observations, and write your conjectures.

Calculate angle measures

1. The measure of the base angles of an isosceles triangle was 50º. What was the measure of the third angle?
2. What name would you give to a triangle that measured 90º, 45º and 45º?
3. Name a triangle with sides 2 cm, 3 cm, and 4 cm, and angle measures 110º and 45º.
4. What would be the measure of the 3rd angle at # 3?
5. Is it possible to construct a triangle with an angle of 100º and 2 sides of 4 cm? What kind of triangle would this be?
6. Name the triangle with 3 angles of 60º.
7. A triangle measured 4cm, 6 cm, and 12 cm with 3 angles < 90º. This is (a) an acute triangle (b) a scalene triangle (c) an obtuse triangle (d) a scalene acute triangle.
8. In an isosceles triangle the angles opposite the equal sides are (a) smaller (b) congruent (c) similar (d) larger
9. Any angle that is > 90º is (a) obtuse (b) scalene (c) acute (d) right
10. There could only be 1 right angle in a triangle. T/F
11. If a triangle has an obtuse angle, then other angles must be acute. T/F

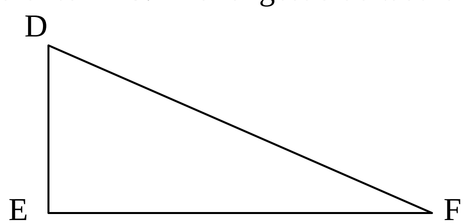
12. A right triangle has 2 acute angles also. T/F
13. One triangle that cannot be drawn in (a) Right scalene (b) Right isosceles (c) Right obtuse (d) Obtuse scalene
14. One triangle that cannot be drawn is (a) Acute equilateral (b) Acute isosceles (c) Acute Scalene (d) Acute obtuse
15.



Angle A = 85º; angle B = 45º; angle C = ?

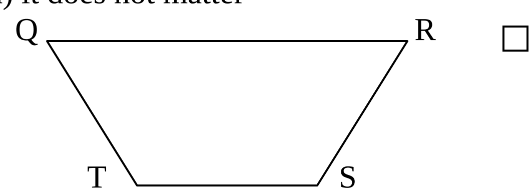
____

16. Refer to # 15. The longest side would be (a) AB (b) BC (c) AC (d) it is equilateral
17.



Angle D = 40º; the hypotenuse = 12 cm; Side DE is 5 cm. Name the type of triangle. ____________________

18. Refer to # 17. Find the measure of angles E and F.
19. Refer to # 17. The largest angle is (a) opposite DE (b) opposite EF (c) opposite DF (d) it does not matter
20.

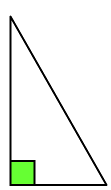


If I drew a diagonal from T to R, triangle QRT ***could*** be (a) acute (b) scalene (c) larger in area than RTS (d) all of the above.

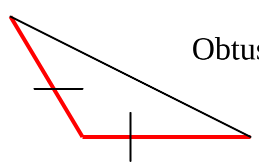
Lesson 4-B solutions

Construction activity:

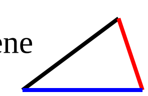
1. Right scalene



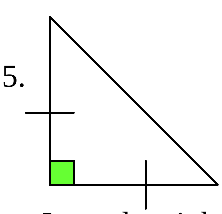
2. Obtuse isosceles



3. Acute scalene

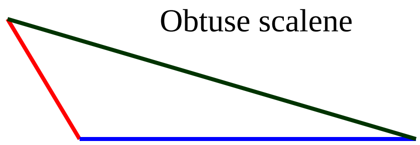


5.

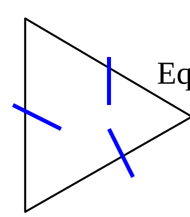


Isosceles right

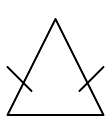
4. Obtuse scalene



7. Equilateral



6. Acute isosceles



Inductive activity
Observations:
Triangle 1: 4 cm, 5 cm, and 6 cm could be constructed
Triangle 2: 3 cm, 5 cm, and 10 cm could not be constructed
Triangle 3: 5 cm, 8 cm, and 10 cm could be constructed
Triangle 4: 5cm, 7 cm, and 9 cm could be constructed
Why is this?
Add any 2 sides and compare the length with that of the 3rd side.
Conjecture: **The sum of the lengths of any 2 sides must be greater than the length of the 3rd side.**

Try triangle 1: 4 + 5 = 9 and 9 > 6; 4 + 6 = 10 and 10 > 5; 5 + 6 = 11 and 11 > 4
Try triangle 2: **3 + 5 = 8 and 8 < 10**; 3 + 10 = 13 and 13 > 5; 5 + 10 = 15 and 15 > 3

One of the equations will not prove the conjecture.

Try triangle 3: 5 + 8 = 13 and 13 > 10; 8 + 10 = 18 and 18 > 5; 5 + 10 = 15 and 15 > 8

On your own do triangle 4.

This is called the triangle Inequality Theorem

Calculate angle measures:

1) 80º	2) Isosceles right	3) Scalene obtuse	4) 25º
5) Yes, Obtuse isosceles	6) Equilateral	7) (d)	8) (b)
9) (a)	10) T	11) T	12) T
13) (c)	14) (d)	15) 50º	16) (b)
17) Right scalene	18) E = 90º; F = 50º	19) (c)	20) (d)

LESSON 4-C
CONGRUENCY AND SIMILARITY

There are 4 bases for determining congruency: Side- side-side (SSS); side-angle-side (SAS); angle-side-angle (ASA); angle-angle-side (AAS).

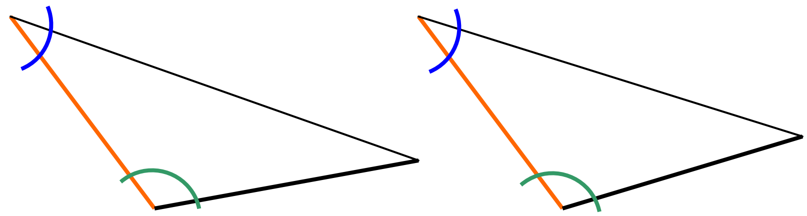
SSS

3 sides of one triangle are congruent to 3 corresponding sides of another triangle. The measure of sides could be stated or symbols used.

SAS

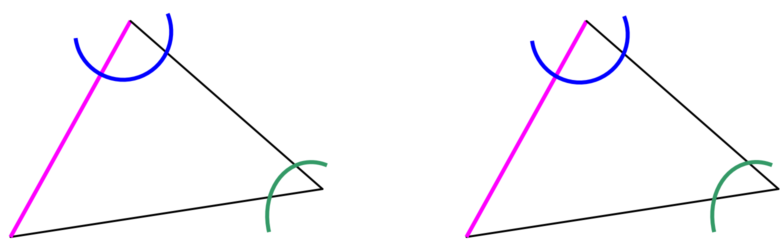
If two sides and an angle between them are equal to 2 sides and the angle between another triangle, both triangles are congruent. The angle between the congruent sides is sometimes referred to as the included angle.

ASA



If 2 angles and an included side are equal to the corresponding angles and included side of another triangle, the triangles are congruent.

AAS



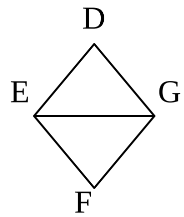
If 2 angles and a side away or non-included side are equal to 2 corresponding angles and a corresponding non-included side, the triangles are congruent.

If triangles are proven congruent by any of 4 rules, then these triangles are identical in all respects. Example: If 2 triangles are congruent by SAS, then all the angles and all the sides of those triangles are congruent. **As long as triangles are congruent by 3 of the required elements, all six elements are congruent.**

Sometimes there is not enough information to prove congruency. In such cases we must say **NOT ENOUGH INFORMATION** or **CANNOT BE DETERMINED.**

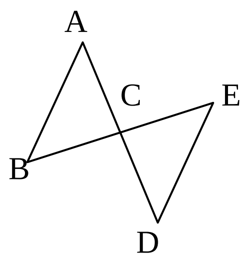
Consider the following examples

1.



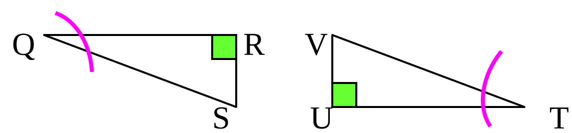
ED = GF; EF = GD.
Is triangle EDG congruent to GFE?

2.



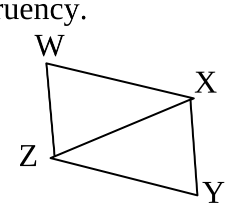
AD = BE; C is at the mid-point of BE and AD. Is triangle ABC congruent to triangle DEC?

3.



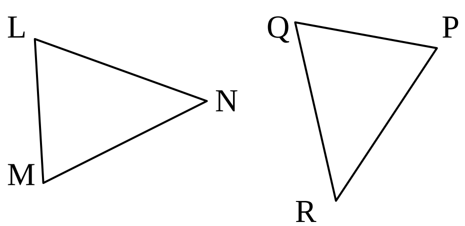
QS = TV; Angle Q= Angel T. Use this information to determine the basis for congruency.

4.



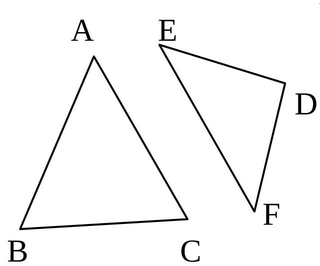
If XZ divides this quadrilateral in to 2 pieces and WXYZ is a parallelogram, is WXZ congruent to YZX?

5.



LM = 4 cm; angle M = 65^{o}; angle N = 60^{o}; PQ = 4cm; angle R = 65^{o}. Are these triangles congruent?

6.



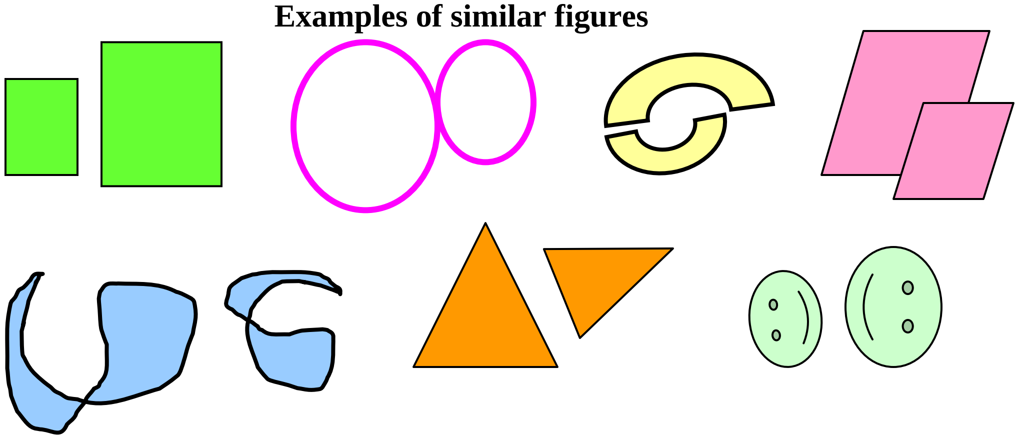
Side AC = EF; AB = AC; Angle E = Angle F; Angle B = angle C. Are these triangles congruent?

When are polygons similar?

Polygons are similar when their ratios are equivalent. They are the same shape but not the same size. Similar polygons have fascinating properties:

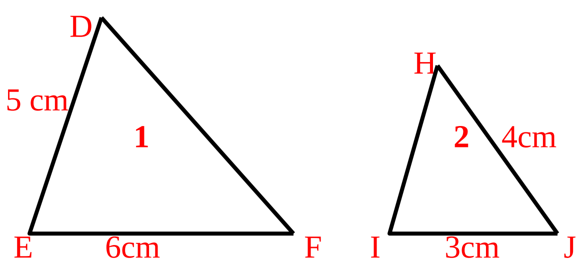
- Their angles are congruent
- Their cross products are equal
- They are dilations of each other; one is larger or smaller than the other by a factor that is not equal to one ($\neq 1$; > 1 or < 1)

Examples of similar figures



Three of these are polygons; the others are not!

Calculations with similar polygons



Triangle 1 is similar to, and larger than, triangle 2. What is the measure of DF and HI?

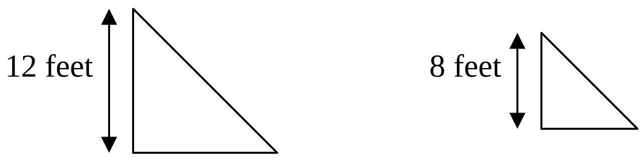
- Compare corresponding sides
- Equivalent ratios should be equal.

The corresponding sides are: DE and HI; EF and IJ; DF and HJ
EF is 2 times larger than IJ; this proportion would be evident on all corresponding sides.

If HJ = 4 cm, then DF = 8 cm; if DE = 5 cm, then HI = 2.5 cm
Compare ratios: $\frac{6}{3} = \frac{8}{4} = \frac{5}{2.5} = \frac{2}{1}$; the ratio of all the corresponding sides is 2:1

NOTE: The important thing is to match all corresponding sides.

More calculations with similar polygons



A B

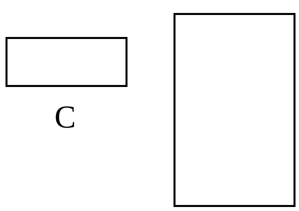
Triangles A and B are isosceles right triangles. Triangle B is $^{2}/_{3}$ the size of triangle A. The height of A is 12 feet; the base of B is 8 feet.
(a) Find the area of both triangles.
(b) What is the **ratio of A to B**?
(c) What is the **ratio of the areas of A to B**?
(c) Find the measure of the hypotenuse of B.

Both triangles are isosceles right; both legs are the same measure. The height of A = 12 feet, then the height of B = $^{2}/_{3}$ of 12 = 8 feet. The base of B = 8, then the base of A = 12 feet.
Area of A = ½ (bh) = ½ (12 · 12) = 72 sq feet
Area of B = ½ (bh) = ½ (8 · 8) = 32 sq feet

Ratio of A to B = 2:3
Ratio of areas of A to B = 72:32 = 9:4 (3^2: 2^2) Observe the pattern

Hypotenuse of B: $A^2 + B^2 = C^2$
$8^2 + 8^2 = 128$ sq feet
$C^2 = 128$ sq feet: $C = \sqrt{128} = (\sqrt{2})(\sqrt{64})$
$= \sqrt{2}\ (8)$ feet or 11.3 feet

More on finding hypotenuse in the unit on Pythagoras' Theorem



C D

C and D are similar rectangles
The length of the longer side of C is 5 cm. The length of the longer side of D is 20 cm. The shorter side of C is 3 cm.

(a) How much bigger is D than C?
(b) Find the area of C and D
(c) Find the ratio of C to D and the ratio of the area of C to D.
(d) Find the perimeter of both rectangles.
(e) Find the ratio of the perimeter of both rectangles

D is 4 times bigger than C.
Area of C = bh = 5 x 3 = 15cm^2; Area of D = 20 x 12 = 240 cm^2
Ratio of C to D = 1:4; ratio of the areas of C to D = 15:240 = 1:16 (1^2: 4^2)
Perimeter of C = 2(5 + 3) = 16 cm; Perimeter of D = 2(20 + 12) = 64 cm
Ratio of perimeter of C to D = 16:64 = 1:4

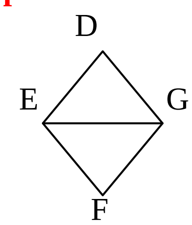
Study the examples, list your observations, and create conjectures about the ratios of the perimeter and area of similar polygons.

Writing 2-column proofs

Some students prefer to write direct proofs of congruency in a 2-column chart or table:

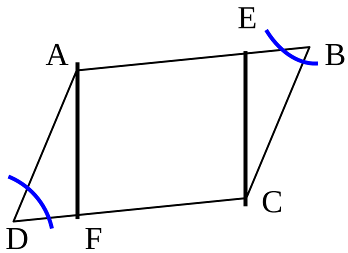
Examples:

.



ED = FG; EF = GD. Is triangle EDG congruent to GFE?

Triangles EDG and GFE	Reasons
1. ED = FG	Given
2. EF = GD	Given
3. EG = EG	Same side or reflexive side
EDG ≅ GFE	SSS (Side, side, side)



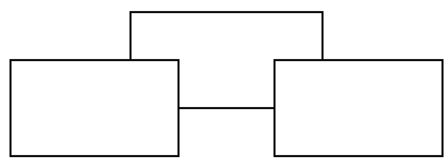
ABCD is a parallelogram; AF is the height of the parallelogram. Is triangle ADF congruent to triangle CBE?

Triangles ADF and CBE	Reasons
1. AD = CB	Opposite sides of a parallelogram
2. Angle AF D= Angle CEB	Right angle formed by perpendicular
3. Angle D = Angle B	Opposite angles of a parallelogram
ADF ≅ CBE	SAA (Side, angle, angle)

Lesson 4-D
Scale Models

A scale model is a drawing or construction that is larger or smaller than an original. All maps, building plans, automobile prototypes are scale models. There is no piece of paper large enough to draw the exact dimensions, so we use some proportion to create a realistic idea of the finished product.

Examples1:



Assume that this is a floor plan of a wing of a school. The actual size of the school could not be drawn on paper; the model will scale down the size by a specific measurement that will be written on the plans. If each completed room is 20 feet by 24 feet, and the drawing measured 5cm by 6cm, the scale used cm to represent feet. ***The question is how many cm represent how many feet?***

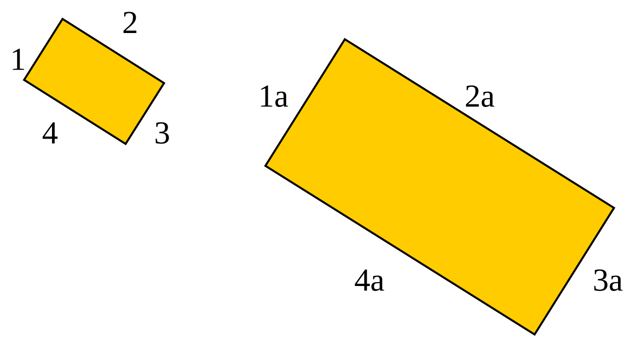
We determine the ratio by comparing the numbers on the longer side and those on the shorter side. We compare 20 and 5, then 24 and 6. We observe that they have 4 as a common factor. **We can conclude that 1 cm represents 4 feet**. Instead of 20 feet we draw 5cm (4 goes into 20 5 times); instead of 24 feet we draw 6cm.

We write the scale like this: **1 cm = 4 feet** or **1cm: 4feet**
Any unit of measure can be used to represent another unit of measure. To return to real measurements we must change back the cm to feet.

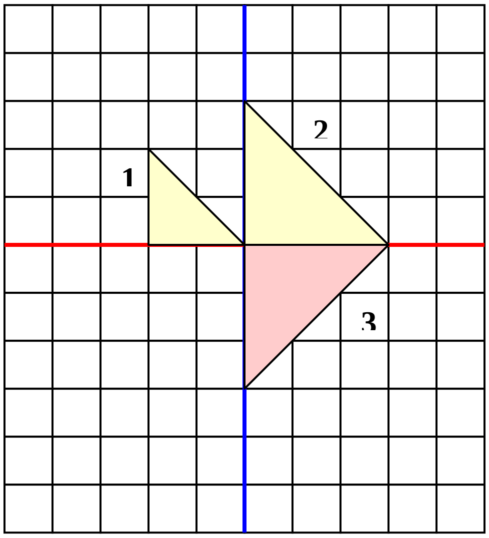
When we draw measurements to scale we actually enlarge or reduce. In geometric terms we dilate the figure. Examples of real life dilations include:

- The pupils of your eyes after medication is applied
- Wallet size or poster size photos
- Maps
- Floor plans and hobby models (cars, planes, trains)
- Clothing of different sizes

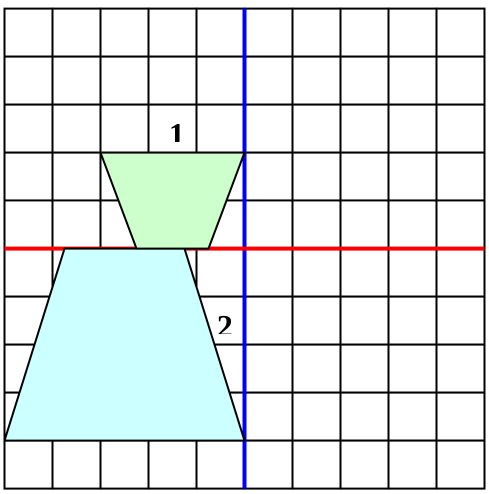
Scale models must contain all the properties of the original: symmetry, corresponding parts, rotation, reflection, and other forms of transformation.



This scale model is twice the size of the original. **Corresponding parts 1a, 2a, 3a, and 4a** match in location to the original.



The original right triangle is dilated twice its size. Compare areas by counting squares in each one. The first has 2 square units; the second has 4 square units. Figure 3 is a 90º rotation and a dilation of the original.



The original trapezoid is dilated and reflected. If each ¼ inch square represents 1 meter, find the measure of $base_1$, $base_2$, and the height of both. There is also symmetry in this set.

Compare their areas. Find the ratio of the perimeter of the first to the second. Write a conjecture about the ratios of the areas of similar figures. **Compare your conjecture in this activity with the one at the bottom of page 32.**

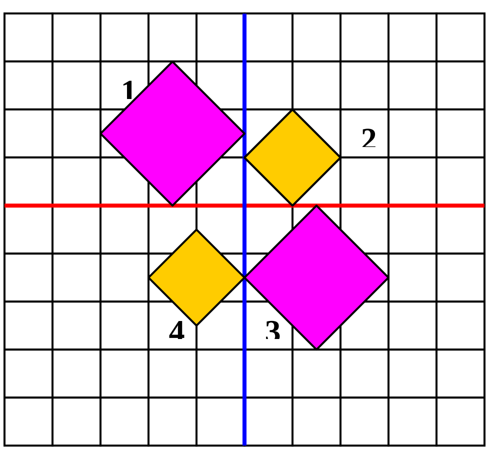
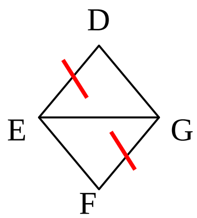


Figure 1 is reduced to about $^1/_3$ its size. Figure 2 is a glide transformation of the original. Figure 3 is another glide transformation that reflects. Figure 4 reflects, glides, and has symmetry. Because of the shape, we can say they rotate.

Solutions to activity on page 34

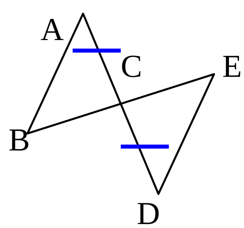
1.



ED = GF; EF = GD.
Is triangle EDG congruent to GFE?

ED = GF; EF = GD; EG = EG (same side, reflexive)

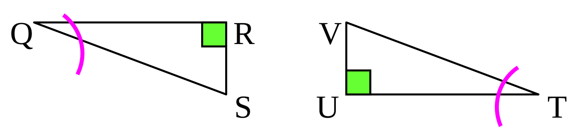
2.



AD = BE; C is at the mid-point of BE and AD. Is triangle ABC congruent to triangle DEC?

AC = DC (mid-point divides the segment equally)
BC = EC (mid-point divides the segment equally)
ACB = DCE (vertical angles)
Triangle ABC ≅ DEC (SAS)

3.

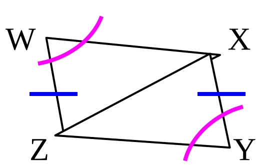


Determine the basis for congruency given the following:

QS = TV (given); Angle R = Angle U (right angles; given); Angle Q = Angle T (given)

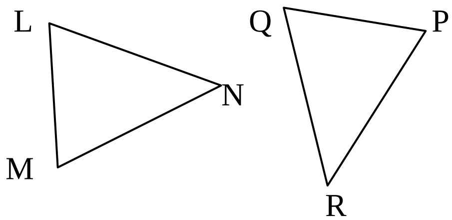
Therefore triangle QRS is congruent to triangle TUV because of (AAS).

4.

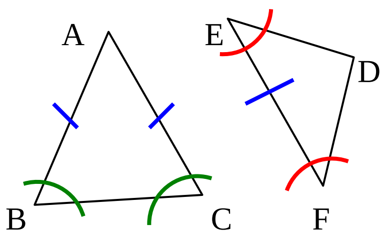


If XZ divides this quadrilateral in to 2 pieces and WXYZ is a parallelogram; is WXZ congruent to YZX?

A parallelogram has opposite sides and opposite angles equal.
WZ = YX (opposite sides of a parallelogram)
W = Y (opposite angles of a parallelogram)
XZ = ZX (same side; reflexive); WXZ ≅ YZX (SAS)
Congruency could also be proven by SSS (WX = YZ: opposite sides of a parallelogram)

5. 

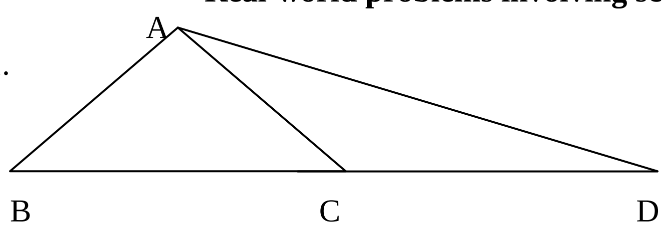
LM = 4 cm; angle M = 65º; angle N = 60º; PQ = 4cm; angle R = 65º. Are these triangles congruent?
LM = PQ (given); M = 65º and R = 65º angles not in corresponding locations; N = 60º; no other information given. There is not enough information to determine congruency.

6. 

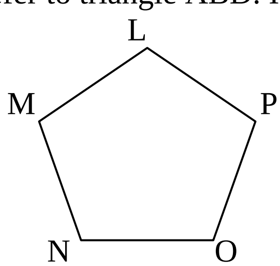
Side AC = EF; AB = AC; Angle E = Angle F; Angle B = angle C. Are these triangles congruent?

The symbols show information about each triangle but little information about corresponding parts of both triangles. Therefore congruency cannot be determined.

Real-world problems involving scale drawing

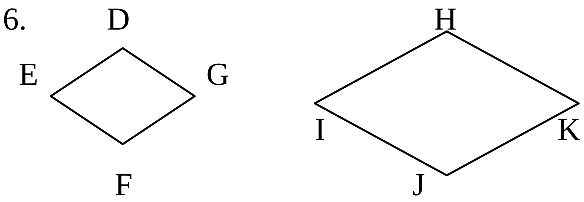
1. 

ABC is an isosceles triangle. AC is a median of ABD. What is the ratio of BC to CD?

2. Refer to triangle ABD. If BC = 5 cm, what is the measure of CD?
3. 

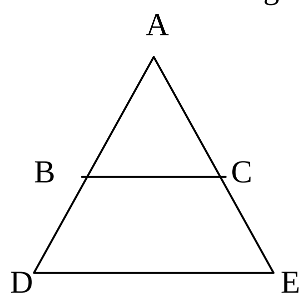
LMNOP is a regular pentagon. A similar pentagon QRSTU has sides in the ratio of 3 to 1. What is the ratio of LM to QR? If UT is 2.5 inches, what is the measure of PO?

4. Refer to #3. The height of one triangle inside LMNOP is 3 inches. What is the area of LMNOP? What is the ratio of the area of LMNOP to QRSTU?
5. Refer to #3. Find the perimeter of LMNOP. Compare that to the perimeter of QRSTU and state the ratio.

6. 

These are both parallelograms.

Angle E is ≡ Angle I; Angle K = 65º; HK = 12cm; DG = 3cm; ED = 2.5 cm.
Find the perimeter of both parallelograms. Find the ratio of the smaller to the larger and state whether they are similar. Consider the theory of the areas of similar figures. State the ratio of the larger to the smaller without calculating.

7. 

ABC and ADE are isosceles triangles. BC is a median and parallel to DE. Find two pairs of equal angles; two pairs of corresponding angles; two pairs of alternate angles; one pair of exterior angles; an exterior angle that is equal to 2 remote interior angles; 2 similar triangles.

8. Write T/F after each statement and provide an explanation for your choice:

a) All equilateral triangles are similar
b) All isosceles triangles are similar
c) All rectangles are similar
d) All parallelograms are similar
e) All trapezoids are similar
f) All regular hexagons are similar
g) All rhombi are similar
h) All congruent figures are similar
i) All similar figures are congruent
j) Two polygons can have the same measure of angles and not be similar

Solutions on page 59

LESSON 5
MEASUREMENT FORMULAS

Lesson 5-A
Perimeter and Area

In this unit there will be many opportunities to practice real-world problems involving perimeter and area of polygons and circles, and volume of 3-D figures.

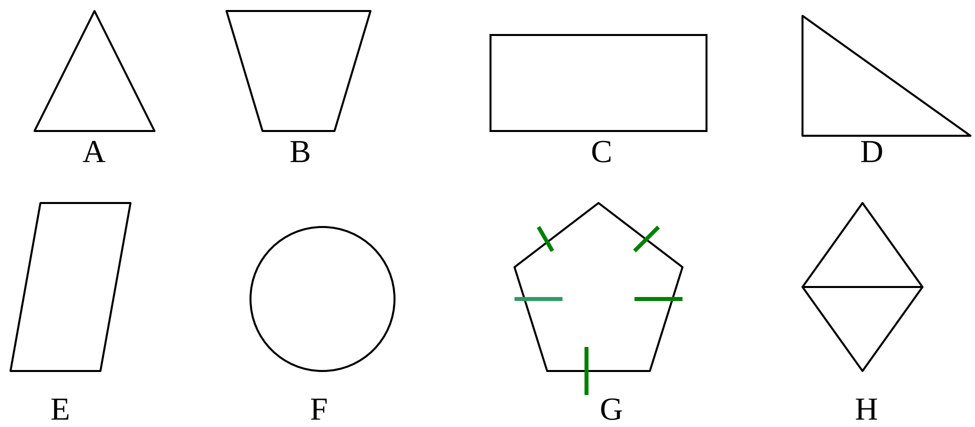
Review of basic formulas

1. Perimeter of a rectangle = 2(l + w) or 2(b + h) or 2l + 2w or 2b + 2h
2. Perimeter of a square = 4s
3. Perimeter of any polygon = sum of all sides
4. Circumference of a circle = π d or 2 π r
5. Diameter of a circle = 2r (r = radius)
6. Radius of a circle = ½ d (d = diameter)
7. Area of a rectangle = bh or lw
8. Area of a square = s^2
9. Area of a trapezoid = ½ (b_1 + b_2) h
10. Area of a parallelogram = bh
11. Area of a triangle = ½ (bh)
12. Area of a regular polygon = area of a triangle x number of triangles or ½ (bh) n
13. Area of a cylinder = Area of the base and top plus area of the rectangle forming the cylinder. The area of the rectangle forming the circular face = circumference multiplied by the height. Remember that a cylinder is made up of a rectangle and two circles.
14. Area of a circle = πr^2
15. Area of a semicircle = ½ (πr^2)
16. Area of a cube = Area of 1 face times 6
17. Area of a rectangular or triangular prism = Sum of the area of each face
18. Area of a triangular or square pyramid = Sum of the area of each face
19. Volume of a rectangular prism = L · W · H or L · W · D (d = depth)
20. Volume of a cylinder = Area of the base times height or depth

Perimeter and Area Problems

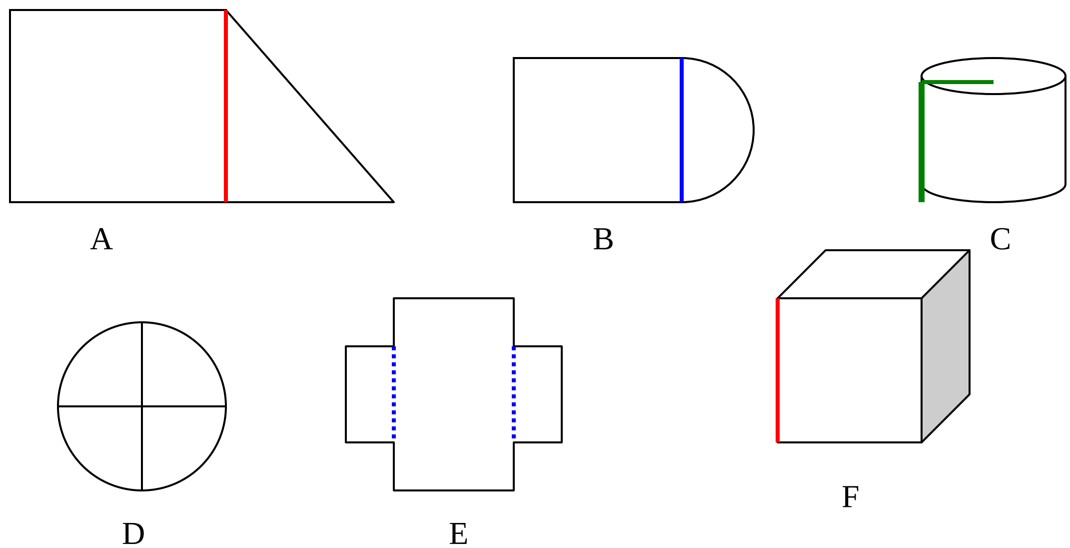
Concept and definition

1. Perimeter means distance around. T/F
2. Area is the measure of the surface. T/F
3. Area is one-dimensional. T/F
4. Perimeter is one-dimensional. T/F
5. Area multiplied by height equals volume. T/F
6. Area is cubic measure. T/F
7. Volume is square measure. T/F
8. Linear measure is one-dimensional. T/F
9. Volume divided by area equals linear measure. T/F
10. Linear measure multiplied by linear measure equals square measure. T/F



11. Name each figure and state the formula for finding area.
12. If the height of A is 7cm and the base is 5 cm, what is its area?
13. The measure of the shorter parallel side is 8 inches and that of the longer side is 10 inches. The height of that figure is 12 inches. What is the area?
14. If the non-parallel sides of B are 9 inches, find its perimeter.
15. Find the area and perimeter of C if the sides are 4 and 6 meters.
16. The base of the right triangle is 7.5 cm and the height is 5 cm. If the hypotenuse is twice as long as the height, find the perimeter.
17. The base of E is one third of its height. If the base is 3.5m, find the area.
18. If F has a radius of 10 cm, find the diameter, the circumference, and the area.
19. G is a regular pentagon. Each side is 8 cm. Find the perimeter.
20. H is a parallelogram. What shape uses the same area formula?

Complex perimeter and area problems



In order to find perimeter or area any of these figures we must recognize that each of these is composed of more than one simple polygon.

A is made up of a rectangle and a triangle. The **height** of the triangle is the same measure as one side of the rectangle. Figure A is also a **right trapezoid**.

B is made up of a rectangle and a semicircle. The diameter of the semicircle is the measure of one side of the rectangle.

C is a cylinder; two circles form the base and top and a rectangle forms the circular face. The **height of a cylinder** is the length of the rectangle that curves to form the circular face. The radius of the circle goes from the center to the circumference. The diameter is the measure across the top passing through the center.

D is a circle divided into 4 parts. There are two diameters that intersect at the center. One can find the circumference or area of a **semicircle** or **one quarter of a circle**.

E has two broken lines to show that two rectangles make up this **irregular dodecagon**.

F is a cube with six square faces. The rectangular prism is constructed in the same way. Care must be taken to get accurate measures for each face when finding area.

Problems

1. Which is the best measurement formula to use to find the area of figure A?
2. If base$_1$ is 5 ¼ feet, base$_2$ is 7 ½ feet, and the height is 4 feet, find the area of A.
3. What would be the distance around figure B if the radius is 21m and the width of this portion of an athletic track is 75m?
4. If the diameter of figure D is 100 cm, what would be the area of ¼ of it?
5. The height of C, a cylindrical water tank, is 4m and its radius is 1m. How many gallons would paint it if 1 gallon paints 9 sq meters? The bottom would not be

painted. What would it cost at \$45 per gallon? (Paint could be purchased in ½ gallons)

6. The height of the cube is 2 feet. Find its perimeter and area.
7. Figure E is part of a company's logo. Each side must be a multiple of 2 cm. The longer sides are double the shorter sides and the shorter sides are 2 cm. How many squares would outline it and how many would fill it?
8. Check the formula for volume of a cylinder. Do we have enough information to find volume? If so what is it?
9. What is the volume of the cube?
10. Divide A into 2 triangles. Find the area of each triangle. Compare the sum of the area of the two triangles with your answer for problem number 1. Justify your answer in words.
11. Imagine you were able to fold the cube from a shape cut out on a sheet of paper. Draw what it would look like. Use the drawing to show that area could be base multiplied by height.
12. Draw the cylinder as surface measure only and show its dimensions.

Solutions and partial solutions to page 44

1. T 2) T 3) F 4) T 5) T 6) F 7) F 8) T 9) T 10) T
11. A = triangle ½ **(bh)** B = trapezoid **½ ($b_1 + b_2$) h** C = rectangle **bh or lw**
D = right triangle **½ (bh)** E = parallelogram **bh** F = circle **πr^2**
G = regular pentagon **½ (bh) n** H = parallelogram, kite, or rhombus **bh**
12. 17.5 cm^2 13) 108 ins^2 14) 36 inches 15) 24m^2 16) 225 cm 17) 36.75m^2
18. d = 2r c = π d area = πr^2 = 3.14 · 10 · 10 = 314 cm^2
19. Perimeter = 8 (5) = 40 cm. 20) E and H would use the same formula.

Solutions and explanations to pages 45 and 46

1. ½ ($b_1 + b_2$) h 2) ½ (5 ¼ + 7 ½) 4 = 25 ½ ft^2

3) Distance around track = one half the circumference plus the distance around the 3 exposed sides of the rectangular part of the track. R = 21m; d = 42m.
½ c = ½ d π = ½ · 42 · $^{22}/_7$ (choose $^{22}/_7$ for pi when the numbers are multiples of 7)
= 66m; 3 sides of rectangular part of the track = 75 + 75 + 42 = 192m + 66m = 258 m

4. A = πr^2 = 3.14 · 50 · 50 · .25 (¼ of the circle) = 1962.5 cm^2

5. Area of the top = 1 · 1 · 3.14 = **3.14 m^2**; Area of circular face = circumference · height
= c = d π = 2(3.14) → 2(3.14) (4) = **25.12 m^2**; add both areas
3.14 m^2 + 25.12 m^2 = **28.26 m^2**
1 gallon paints 9 sq. m; 28.26 ÷ 9 = 3.14 gallons; go to **3.5 gallons**. 1 gallon costs \$45.00, 3.5 gallons cost \$45 x 3.5 = **\$157.50**

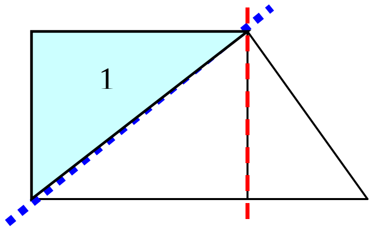
6. Perimeter of the cube is the distance around the edges. A cube has 12 edges and 6 faces. Length of 1 edge = 2 cm. Length of 12 edges = 24 cm.
Area of each face = 2 x 2 cm = 4 cm^2; area of 6 faces = 6 (4) cm^2 = 24 cm^2

7. The outline of a figure is the perimeter. The fill-in is the area. There are 4 long sides each 4 cm and 8 short sides each 2 cm. The perimeter is 32 cm.
Area of 1 large rectangle = 4 cm (8 cm) + area of 2 small rectangles (2 x 4cm x 2)
32 cm^2 + 16 cm^2 = 48 cm^2

8. There is enough information to find the volume of the cylinder. V = A (h) where
A = Area of the top and h = height. V = AH = 3.14 m^2 x 4 m = 12.56 m^3

9. Volume of the cube = LWH = 2^3 = 8 ft^3

10.



The dotted line divides the trapezoid into 2 triangles. The dotted red line is the height of both triangles that are part of the trapezoid.

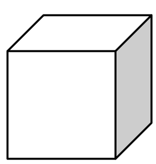
The base of the triangle (triangle 1) = 5 ¼ feet; the base of triangle 2 = 7 ½ feet; the height = 4 feet; Area formula = ½ (b^1 + b^2) h;
Area of each triangle = ½ bh of triangle 1 + ½ bh of triangle 2
Compare both formulas; look for what's common and factor. **½ and h** are common factors. Put those **outside the parentheses** and put the rest of the inside. Use the distributive property.
½ ($base_1$ + $base_2$) **h** = ½ b**h** of triangle **1** + ½ b**h** of triangle **2**

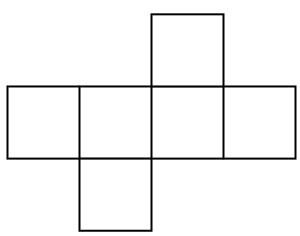
Let's work it to prove whether this works!

½ (5 ¼ + 7 ½) 4 = ½ (5 ¼ · 4) + ½ (7 ½ · 4)
½ (12 ¾)4 = ½ (21) + ½ (30)
25 ½ sq ft = 10 ½ + 15 = 25 ½ sq ft

Justification: The parallel lines are equidistant; this feature inside a trapezoid ensures that the height is a constant. The base of each triangle creates the shape whether it is a right or an isosceles triangle. Using the distributive property shortens the process of finding the area of 2 triangles. The calculation proves that it works.

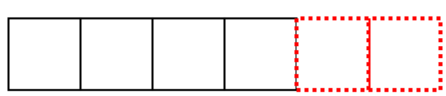
11.





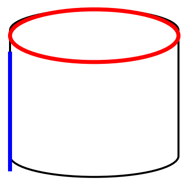
One can draw a cube from this shape

Area = BH; if we string all the cubes next to each other what would happen? Let's see!



The last 2 squares are the ones from the top and the bottom. The height is 2 cm and the width of each square is 2 cm. Count the measure across the top. Multiply that **base** by the **height.** We now have 12 cm x 2 cm = 24 cm^2

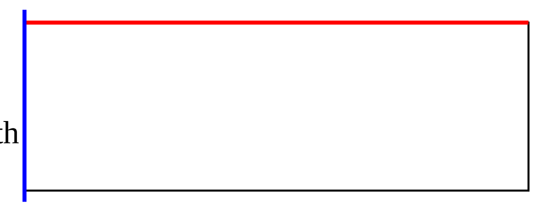
12. The cylinder as surface measure:



The circumference is like a straight line; π d = circumference

π d = c (length)

Height or width



Area of a rectangle = LW

Area of a cylinder = Circumference (**π d**) multiplied by height **plus** area of the top and bottom (**$2\pi r^2$**) = $\pi d \bullet h + 2\pi r^2$

Interesting links

http://www.math-prof.com/Geom/Geom_Ch_26.asp

http://www.aaamath.com/g79-surface-area-cylinder.html

http://www.aaamath.com/B/g88_vox5.htm

(All websites Accessed: 1/23/2006)

Lesson 5-B
Measurement on the coordinate plane

In this lesson we will use several formulas as we work on the coordinate plane. We will compare what we do with and without graphing. This is one of the most important sections of the geometry curriculum; it revisits the algebra-geometry connection in an attempt to re-unify these two branches of mathematics.

Formulas

Slope: Slope is represented by an algebraic and a geometric formula. The simplest way to express slope is **rise/run**. This is clearly observed on a graph and can be determined by counting squares. The Algebra 1 curriculum has an extensive lesson on slope that can be reviewed. In that curriculum an equation called the **slope-intercept format** is used. **Y = mx + b** contains the **slope**, the **y-intercept**, and a way to determine the **coordinates** in a table of values. The value of ***m*** or the **coefficient of x** provides the slope and the value of ***b*** gives the y-intercept or the point where the line crosses the y axis. If we have coordinates we can substitute their values to determine slope by applying the formula

$$\frac{y_2 - y_1}{x_2 - x_1}$$

http://math.about.com/library/blslope.htm

The link provides more details and an interactive lesson.

Mid-point: The mid-point is the **point halfway between the endpoints of a line segment.** Sometimes we must determine the mid-point from a table of values or on a graph. Errors are possible because scales may not indicate .the real values. The mid-point formula will reduce errors.

$$\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

Distance: Substituting values in a formula will allow us to quickly find out the distance between two points on a graph, or between 2 sets of values on a coordinate plane. Distance is much like using the Pythagorean Theorem to get **leg C** (the hypotenuse) of a right triangle. It looks more menacing than it is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_1 - y_2)^2}$$

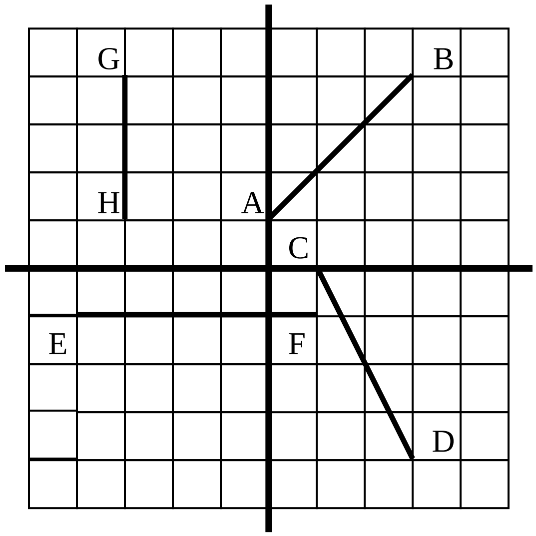
$$C^2 = a^2 + b^2$$

$$C = \sqrt{a^2 + b^2}$$

(x_1, y_1) and (x_2, y_2) represent the coordinates of any two points on a plane or in a table of values.

Examples

Find slope by using rise/run:
Assume that the intervals are units of 1 on both X and Y axes.
Rise is vertical; run is horizontal. Count the number of squares from the beginning of the line to the end. Use the letters of the alphabet in sequence.



AB = rise (3 units up); run (3 units across) = 3/3 = 1
CD = rise = (4 units down); run (2 units across = -4/2 = -2 (down is negative)
EF = rise = 0; run = 5 (5 units across) = 0/5 = 0
GH = rise = -3; run = 0 = undefined
When the numerator is zero, there is no slope; when the denominator is zero, the slope is undefined.

Find slope using the y-intercept format: y = mx + b
1. Y = 4 2) y = x + 2 3) y = 3x – 2 4) y = -2x + 1 5) y = x – 0

1. **Y = 0x + 4. Slope = 0.**
2. Slope = 1; 3) slope = 3 4) slope = -2 5) slope = 1

Find the y-intercept in each of the equations (1-5):

1. y = 4 for any value of x; the line does not slope; it is horizontal; see example EF where y = -2 for all values of x. The y-intercept = 4.
2. y-intercept = 2; the line crosses y at y = 2
3. y-intercept = -2; the line crosses y at y = -2
4. y-intercept = 1; the line crosses y at y = 1
5. y-intercept = 0; the line crosses y at the origin

As long as an equation is given, one can identify the slope and the y-intercept.

Write an equation from the y-intercept values and m:

1. m = 2; y-intercept = 0 2) m = -1; y-intercept = 3 3) m = 4; y-intercept = -5

Solutions: 1. y = 2x + 0 2) y = -x + 3 3) y = 4x – 5

Write an equation from the given conditions:

1. m = ½ through a point at (3, **2**) 2) m = -3 through a point at (0, **-1**)
3. m = 2 through a point at (-1, **4**) 4) m = 5 through a point at (1, **2**)

Solutions: 1) y = ½ x **-1** 2) y = -3x **– 1** 3) y = 2x **+ 2** 4) y = 5x **- 3**

Re-write the equation in the y-intercept format:

1. 5x + y = 3 2) -3x + y = 6 3) 4 = y – 2x 4) 3x = y + 5

Solutions: 1) y = -5x + 3 2) y = 3x + 6
3) –y = -2x – 4 → y = 2x + 4 4) –y = -3x + 5 → y = 3x - 5

Determine coordinates in a table of values from an equation:

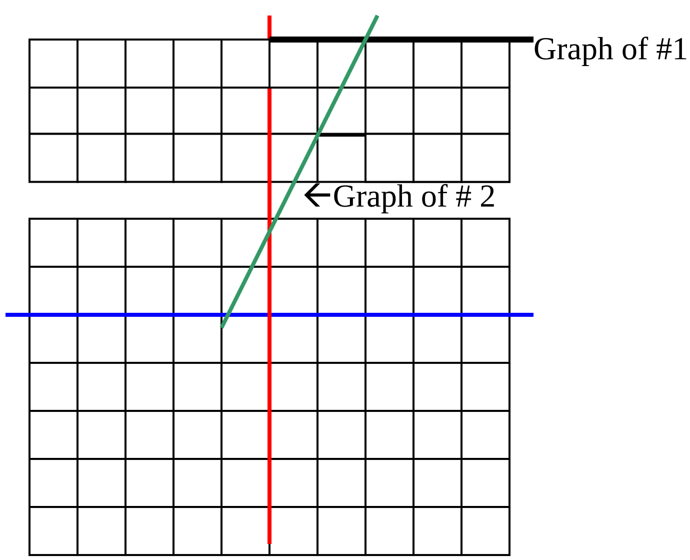
1. y = 5 2) y = 2x + 1 3) y = -3x + 2 4) y = ½ x + 4

Substitute a sequence of your own values for x and calculate the value of y

x	y	x	y	x	y	x	y
0	5	-1	- 1	-1	5	0	4
1	5	0	1	1	-1	2	5
2	5	1	3	3	-7	4	6
3	5	2	5	5	-13	6	7

What do you observe about the slope of each line even before graphing?
1. The slope is a horizontal line; see line EF on page 46.
2. The slope is increasing and the line starts in quadrant 3.
3. Is the slope increasing or decreasing in example 3 and where does the line start?
4. Determine whether the slope is increasing or decreasing and tell in which quadrants the line starts and ends.

The sloping line is a graph of #2. It starts in quadrant 3 and it is increasing.
The slope is 6/3 = 2
The y-intercept is 1.
Observe the equation of the line for each of these values.



Problems involving slope

Determine the slope and y-intercept of the following equations:

1. $y = \frac{1}{2}x + 2$ 2) $x = -3$ 3) $2x – y = 5$ 4) $y = 3x + 0$

Write an equation of the line satisfying the following conditions:

5. m = 2, b = -5 6) m = - 4, b = 4 7) m = -6, b = 5
8. m = ½ , y-intercept = 0 9) m = - 4, y-intercept = -1 10) m = 3, y-intercept = - 2
11. m = -2, through a point at (5, -6) 12) m = 4 , through a point at (½, - ¼)

Graph each equation and find slope by counting squares:

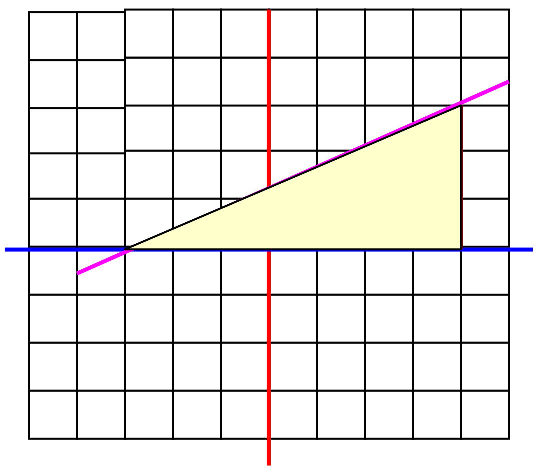
13. $y = x + 4$ 14) $y = 2x – 3$ 15) $y = -2x + 1$

Create a table of values from each equation and find slope using the formula:

16) $y = 4x – 3$ 17) $y = 5x$ 18) $y = 3x - 1$

Mini Project involving Mid-Point, Distance, and Slope

A high school student wanted to design his own skateboard ramp for an engineering competition. He had to complete his design on graph paper showing an appropriate scale using slope, mid-point, and distance formulae, among other things. This is what he had so far; help him complete it.



The ramp had these specifics:

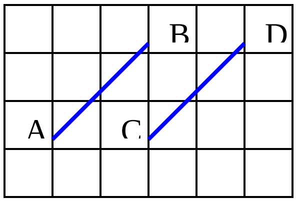
1. Each block should represent no more than 5 feet.
2. The ramp should be between 18 and 30 feet on the ground and no taller than 6 feet.
3. The distance formula must be applied to show the distance on the ground.
4. Use 2 ways to calculate slope.
5. Find the length of the ramp (where the ramp is the hypotenuse of the triangle and $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$. See page 36 for example.
6. Write the equation of the line in the y-intercept form.

Clues you can use

1. Create a scale and interval for the graph. Use the specifics above to guide you.
2. Find the coordinates of the line that creates the ramp. Use those coordinates to find slope; find slope by counting squares.

3. Find the height of the ramp. Once you have the height, use Pythagoras' Theorem to calculate the length of the ramp.
4. If you have slope and the y-intercept, you can create the equation of the line.

Parallel and Perpendicular Slopes

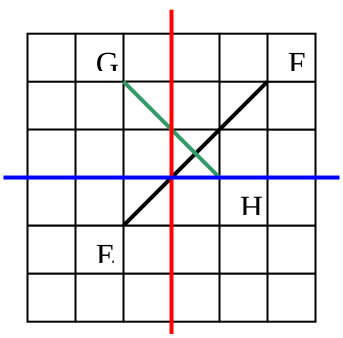


AB is parallel to CD.
Find the slope of AB by counting squares; find slope of CD by counting squares.

The slope of AB = ___________; the slope of CD = _____________

Your answers should be identical. **Parallel lines have the same slope**.

In the following diagram, EF is perpendicular to GH.
Perpendicular lines are a little more complex. The slope of GH is – 2/2 = -1
The slope of EF is 3/3 = 1; multiply the slopes: (- 1) 1 = **-1**; **if the product of the slopes is -1 the lines are perpendicular (multiply by the multiplicative inverse).**



Activities

State the slope of a line running parallel to each ordered pair:

1. (a) (1, 2), (4, -6) (b) (3, -1), (4, 2) (c) (-7, 5), (1, 1)

State the slope of a line perpendicular to each ordered pair

2. (a) (3, 2), (1, 5) (b) (-1, -1), (3, 2) (c) (1, 8), (-2, -3)
3. Describe the slope of a line that is (a) vertical (b) horizontal (c) rises from left to right (d) falls from left to right (e) falls from right to left
4. Create a system of equations or equations of 2 lines that are parallel to each other.
5. Create a system of equations or equations of 2 lines that are perpendicular to each other.

Optional Internet interactive web sites for further practice

http://www.quia.com/jq/47238.html

http://users.stlcc.edu/amosher/ParalellandPerp.htm

(All websites Accessed: 1/23/2006)

Lesson 5-C
Review of the Algebra-Geometry Connection
Graphing Systems of Linear equations and Inequalities

In this lesson we will graph systems of equations and inequalities and also work to solve problems by substitution. One of the important aspects to this lesson is locating or identifying the solution in a graph as well as by substitution. Let's get busy!

Recall:

- The graph of 2 equations ***may*** intersect at some point called the solution
- The graphs of parallel lines do not intersect
- The graphs of inequalities intersect and give a range of answers that are shaded

Example 1: y = 2x; y = -2x + 4. Graph these two first degree equations and identify their point of intersection.

Step 1: Set up a table of values to determine the ordered pairs for each equation.

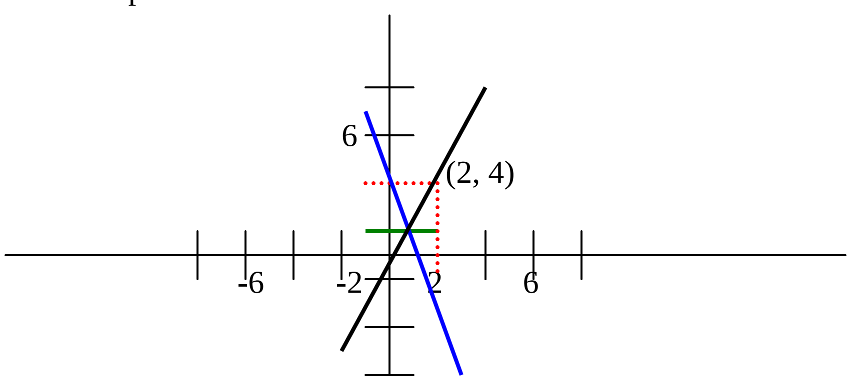
Y = 2x

x	y
2	4
1	**2**
0	0
-1	-2

y = - 2x + 4

x	y	
2	0	
1	**2**	(Solution)
0	4	
-1	6	

Step 2: Graph the coordinates.
Step 3: Read the point of intersection. Observe that the 2 sets of values are the same.



The point of intersection is (1, 2). That point defines the solution.

Quick Practice

Find the ordered pair that is the solution of both graphs:

1. y = 2x + 1; y = -2x + 5
2) y = x – 3; y = 2x – 1
3. y = x + 2; y = 2x – 4
4) y = 2x + 3; y = x – 2

Example 2: Use substitution to solve the system of equations (no graphing).
Y = 2x + 1; y = - 2x + 5

Step 1: Line up the equations; use an operation to eliminate one of the variables.
Y = 2x + 1 (equation 1)
y = -2x + 5 (equation 2) If we add we get rid of 2x
2y = 6; y = 3
Substitute y = 3 into any of the equations
y = 2x + 1; 3 = 2x + 1; **2x = 2; x = 1**
The solution is: (1, 3)

Example 3: Find the ordered pairs to prove that example 2 is accurate (no graphing).

Y = 2x + 1

x	y
1	**3**
0	1
-1	-1

y = -2x + 5

x	y
1	**3**
0	5
-1	7

Quick Practice

Use substitution to find the solution of each system of equations:
1. y = 2x + 1; y = -2x + 5
2) y = x – 3; y = 2x – 1
3. y = x + 2; y = 2x – 4
4) y = 2x + 3; y = x – 2
5. y = 2x + 4; y = 2x
6)y = -3x; y = -2x – 8

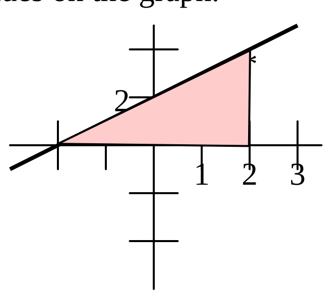
Are they parallel, perpendicular, or neither?
7. y = 3x + 2; y = $-^{1}/_{3}$ x + 10
8) y = 5x – 3; y = 5x + 7
9) y = 2x – 5; y = -1/2 x + 5
10) y = 3x + 1; y = 2x + 1

Review Inequalities

An inequality is a statement that contains a symbol of inequality: $<$, $>$, $\geq$, $\neq$, $\leq$.
The graph of an inequality includes a range of answers.
Example 1: y < x + 2 ——→ **Step 1**: assume that y = x + 2
Step 2: Create a table of values for y = x + 2

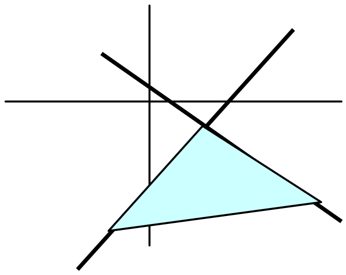
x	y
2	4
1	3
0	2

Step 3: Graph these points. Shade the area below the graph; these values are less than the values on the graph.



Graph Systems of Inequalities

Systems of equations intersect; the graphs of inequalities intersect as well. Your answer looks like this with a shaded area indicating values greater than or less than the solution.



Quick Practice

Find the ordered pairs for each system of inequalities, graph them, and write their solution.

1. y > x + 1; y > - x + 3 2) y < 2x – 5; y < x – 3
3. y > -x + 6; y > x – 2 4) y < x – 6: y < -x + 5

State the ordered pair that satisfies each equation:

5. y = -x/3 + 2 (a) (0, 2) (b) (-1, 4) (c) (6, 0) (d) (-3, 3)
6. 2x – 5y = -1 (a) (0, 5) (b) (2, 1) (c) (0.5, 0) (d) (-2, -1)

Use substitution to solve the system of inequalities:

7. 2y + x > 7; 3y – 2x > 7 8) x + 2y < 8; y < x + 4

Optional Web sites for further practice

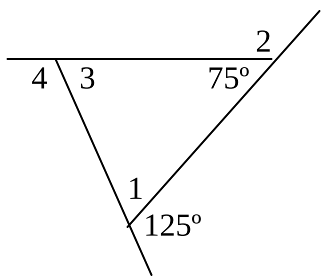
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-02-825178-4&chapter=3&lesson=4

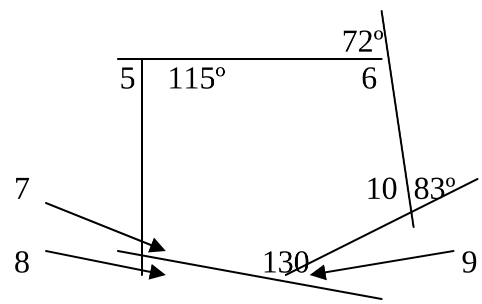
http://cwx.prenhall.com/bookbind/pubbooks/tobey3/medialib/course_notes/ch04_systems_of_equations/inequalities.htm

(All websites Accessed: 1/23/2006)

LESSON 5-D
REVIEW OF LESSONS 4 AND 5

1. Why is the formula for finding the sum of the interior angles of a polygon $(n - 2)180^{\circ}$?
2. How many triangles would there be in a 20-sided polygon and what is the sum of their interior angles?
3. Complete the statement: If we know how to find the sum of the interior angles of a regular polygon, we can also find ______________________________________
4. The measure of an exterior angle of a triangle was 105º. One of the interior remote angles was 75º, what was the measure of the other?
5. Interior angles are complementary to their exterior angles. T/F
6. If #5 is true explain; if it is false replace one word in the sentence to make it true.

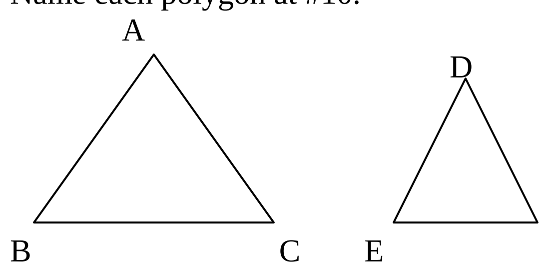




7. Refer to the triangle: Find the measure of angles 1, 2, 3, and 4.
8. Refer to the triangle: Prove that the sum of the 2 remote interior angles is equal to the exterior angle.
9. Refer to the pentagon: Find the measure of angles 5, 6, 7, 8, 9, and 10.
10. Draw or construct polygons that match each description:

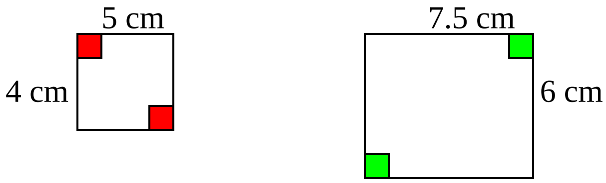
a) 2 congruent sides and 1 right angle
b) 5 congruent sides and 5 congruent angles
c) 2 pairs of parallel sides and 4 right angles
d) 1 obtuse angle and 2 congruent sides
e) 1 right angle and 1 pair of parallel sides
f) 0 right angles, 1 pair parallel sides, and 1 pair congruent sides
g) 0 right angles, 0 parallel sides, and 0 congruent sides
h) A quadrilateral with 0 parallel sides and 2 pairs of adjacent equal sides.

11. Name each polygon at #10.
12.

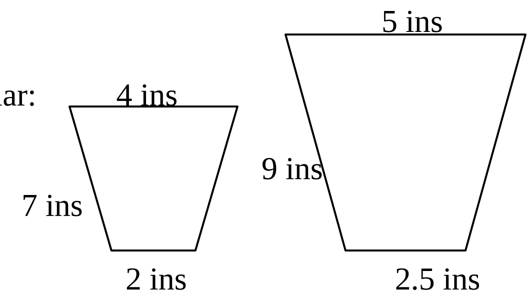


Given: Angle A= Angle D
Angle B= Angle E
Angle C= Angle F
Triangle ABC ~ DEF. State your reasons

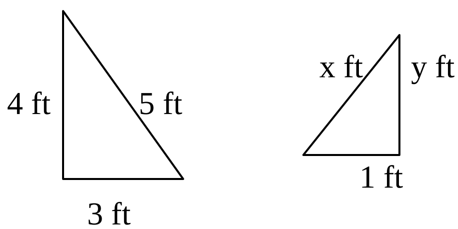
13. Determine whether the figures are similar:



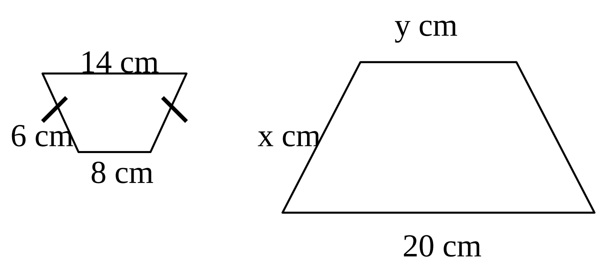
14. Determine whether the figures are similar:



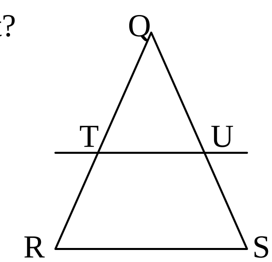
15. Find the values of x and y:



16. Find the values of x and y:



17. Are they similar or congruent?

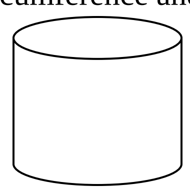


TU is drawn at the mid-point of QR and is parallel to RS. Δ QRS is isosceles. What is the relationship between Δ QTU and QRS? State your reasons.

18. Construct an isosceles triangle on a coordinate plane. Rotate a similar triangle 90º on the same plane. The second triangle must be 2 times the size of the first. List the coordinates of both triangles. What do you observe about the coordinates?

Solutions to real-world problems (1-8) on page 41-42

1. The ratio of BC to CD is 1:1; both are ½ of the segment, BD.
2. The measure of CD = 5 cm
3. The ratio of LM to QR = 1:3. If UT is 2.5 inches, PO = 2.5 ÷ 3 = $.\overline{83}$.
4. The area of LMNOP = ½ (bh) n = ½ · .83 · 3 · 5 = 6.25 inches ^{2}. The ratio of the area of LMNOP to QRSTU = 1:9
5. The perimeter of LMNOP = .83 (5) = 4.15 inches; the perimeter of QRSTU = 2.5 (5) = 12.5 inches; the ratio of LMNOP to QRSTU = 1:3
6. Perimeter of DEFG = 11 cm; perimeter of HIJK = 44 cm. The ratio of the smaller to the larger = 1:4; DEFG ~ HIJK; the ratio of the area of the larger to the smaller is 16:1 or 4^2 to 1^2.
7. Two pairs of equal angles: ABC and ACB; two pairs of corresponding angles: ABC and DBE; two pairs of alternate angles: DBC and ACB; one pair of exterior angles: DBC and ECB; DBC ≡ ACB + BAC; Δ ABC ~ ADE
8. (a) T, same angle measures; (b) F angles may not be congruent; (c) T, angles are congruent; (d) F, angles may not be congruent; (e) F angles may not be congruent; (f) T, angles are congruent; (g) F, angles are not congruent; (h) T, the angles are congruent; (i) F, angles are congruent but side are not; (j) F, if the angles are congruent, the sides are congruent also.

1. Find the area of a triangular pyramid with a height of 17m and 3 base edges of 10m. The bottom has its own surface height of 7 m. (Think of the number of bases and faces in this 3-D figure and draw to make it real)
2. A circle with a radius of 21 feet was divided into 3 equal parts. Find the circumference and area of one part.
3. 

   The diameter of this cylinder was 18 meters and its height 28 meters. What is the area of this cylinder? What was its volume?
4. Use the table of values to find (a) slope (b) distance (c) mid-point (d) equation of the line.

x	y
1	5
2	7
3	9

Solutions to numbers 1, 2, 3 and 4 on this page.

1. Bottom = [(7 x 10) ÷ 2] + 3 faces [3(17 x 10) ÷ 2] = 290 m^2 2) 462 sq feet

3. Area of tank = area of top and bottom + area of circular face = $2 \pi r^2 + d \pi \cdot h$
= (2 x 3.14 x 9 x 9) + 18 x 3.14 x 28 = 508.68 + 1582.56 = 2091.24 m^2; V = 7121.52 m^3

4. **Slope** (pick any two points) = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 5}{2 - 1} = \frac{2}{1}$; **equation of the line**: y = 2x + 3

Distance = $\sqrt{(x_2 - x_1)^2 + (y_1 - y_2)^2} = \sqrt{(2 - 1)^2 + (5 - 7)^2} = \sqrt{1^2 + -2^2} = \sqrt{5} = 2.23$

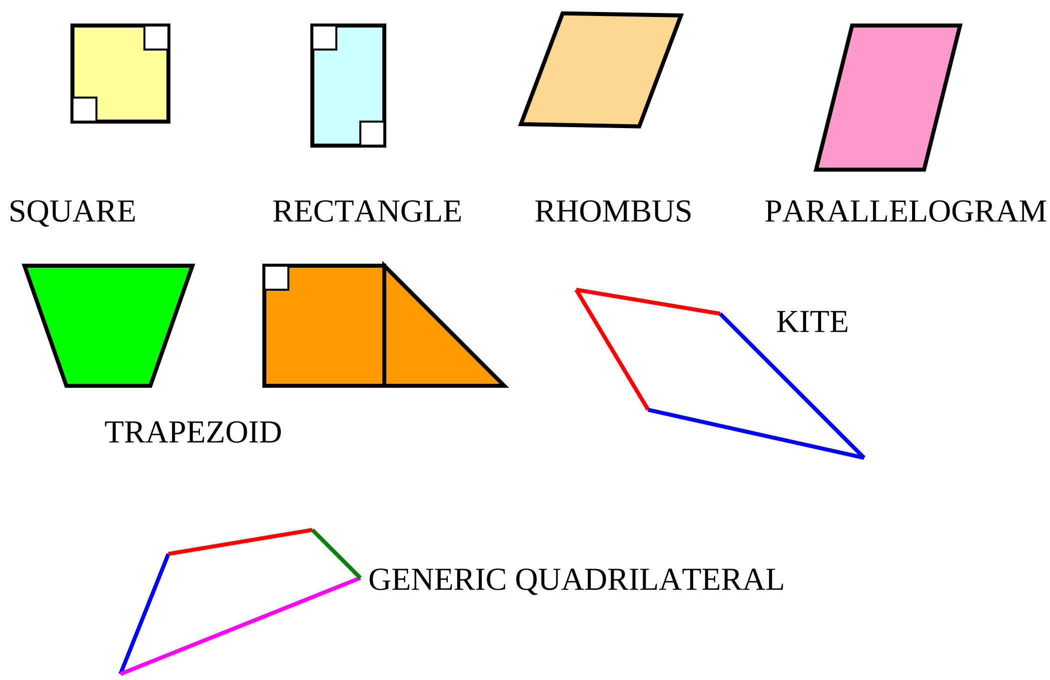
Mid-point = $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} = \frac{1 + 2}{2}, \frac{5 + 7}{2} = \frac{3}{2}, \frac{12}{2}$ = (1.5, 6)

Quadrilaterals in Depth

There are 7 quadrilaterals. Can you name them?

- The **square** or the perfect quadrilateral has all sides and angles congruent and 2 pairs of parallel sides
- The **rectangle** is similar to the square with 4 right angles and 2 pairs of parallel sides but has 2 pairs of congruent sides
- The **rhombus** is also similar to the square with 4 congruent sides but has no right angles; like the rectangle and square the rhombus has 2 pairs of parallel sides
- The **parallelogram** has 2 pairs of congruent sides, no right angles, and 2 pairs of parallel sides
- The **trapezoid** can have 1 right angle or no right angles; a trapezoid may also have one pair of congruent sides; all trapezoids have 1 pair of parallel sides
- The **kite** is similar to a rhombus but has some different properties; it has 2 pairs of consecutive congruent sides, no right angles, and no parallel sides
- The **generic quadrilateral** is unlike any other quadrilateral with no congruent sides, no right angles, and no parallel sides

The sum of the interior angles of any quadrilateral equal 360º



Quadrilaterals with 2 pairs of parallel sides are called **parallelograms**. Which of these are parallelograms?

The **right trapezoid** has 2 right angles; the **isosceles trapezoid** has 2 congruent sides.

How are the square and the rectangle similar? How are they different?
In what ways are the square and rhombus similar; in what ways are they different?
Do the rhombus and the parallelogram have anything in common?
Do the kite and the trapezoid have anything in common?
How is the generic quadrilateral different from other quads?

Activity

1. Get several paper squares of the same size; fold one down the center to create a rectangle.
2. Fold down the corners of another square to shape a parallelogram.
3. Use another square to form a trapezoid.
4. Take another square and make a rhombus. You would be folding smaller corners similar to shaping the parallelogram.
5. Try to create the 2 trapezoids.
6. Finally make the generic quadrilateral.
7. Draw diagonals in each of the quadrilaterals.

Get ruler and protractor, paper and pencil

1. Measure the diagonals of the square and record your results.
2. Measure the angles formed at the intersection of the diagonals and record the results.
3. The diagonals would divide the right angles. Measure each piece. Make a record.
4. Measure the diagonals of the rectangle and record the results.
5. Measure the angles formed at the intersection of the diagonals. Record the results.
6. Measure the new angles created at the corners. Record the results.
7. Continue measuring diagonals and angles at the corners; make recordings with each of the quadrilaterals.
8. Make as many conjectures as possible about the diagonals of each quadrilateral and the angles created at the corners.
9. Knowledge of these properties will facilitate calculation of angle measures and area or perimeter of these quadrilaterals.

Complete the table
Write Yes or No in each column
Compare your conjectures with the responses in the table

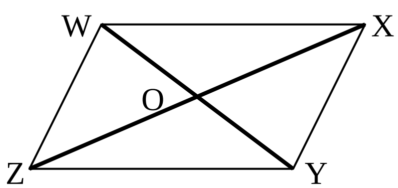
Property	Parallelogram	Rectangle	Rhombus	Square
The diagonals are congruent				
The diagonals bisect each other				
Each diagonal bisects a pair of opposite angles				
The diagonals form 2 pairs of congruent triangles				
The diagonals are perpendicular				
The diagonals form 4 congruent triangles				

Write true or false after each statement:

1. All quadrilaterals have 3 sides
2. Some quadrilaterals are parallelograms.
3. A trapezoid is not a parallelogram.
4. A rhombus is a kite.

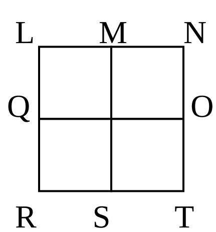
5. A kite is a rhombus.
6. A rectangle is a quadrilateral with 4 right angles. That means that a square is a rectangle.
7. An isosceles trapezoid has 2 pairs of parallel sides.
8. All trapezoids have at least one pair of parallel sides.
9. All trapezoids have exactly one pair of parallel sides.
10. A generic quadrilateral has no similarity to other quadrilaterals except its 4 sides.
11. The diagonals of a square bisect each other.
12. The diagonals of a rectangle bisect each other.
13. Vertical angles form at the intersection of diagonals inside any quadrilateral.
14. The diagonals create pairs of alternate angles inside a parallelogram.
15. The diagonals of a parallelogram do not bisect the opposite angles inside the parallelogram.
16. A quadrilateral EFGH has EF parallel to HG and EH parallel to FG. EFGH is a parallelogram.
17. Refer to #16. EH is congruent to FG but shorter than EF. EF is also congruent to HG.
18. Refer to #16. Diagonal EG is congruent to FH.
19. Refer to # 16. The diagonals EG and FH intersect at O. HO ≠ FO.
20. Refer to #16. Angle EOH is congruent to GOF.

Use your knowledge of quadrilateral properties to solve:



WXYZ is a parallelogram. Diagonals WY and XZ intersect at point O.
ZX is 10 cm and WY is 8 cm.

21. ZO = ____________cm 22) XO = ____________cm 23) WO = ____________cm
24. YO = ____________cm 25) Angle WOZ ≡ ______________
26. Angle WOX ≡ ____________ 27) Angle ZWO ≡ ______________
28. Triangle ZOY ≡ ___________ 29) Triangle ZOW ≡ _____________



LNTR is a square. MS is parallel to LR. M is the mid-point of LN. QO is parallel to RT. QO and MS intersect at P.

30. Name 2 congruent squares and 2 congruent angles.
31. Find the measure of LQO.
32. (a) Is QO congruent to MS? (b) Name 2 pairs of equal supplements.

Solutions to pages 60 and 61

Page 60: The Square and the rectangle both have 4 right angles; the square has 4 congruent sides but the rectangle has 2 pairs of congruent sides.
The square and the rhombus both have 4 congruent sides. The rhombus has no right angles.
The rhombus and the parallelogram are both parallelograms.
The kite and the trapezoid have only the number of sides in common.
The generic quadrilateral is different from all other quadrilaterals: it has no right angles, no congruent sides, and no parallel sides.

Page 61:

Property	Parallelogram	Rectangle	Rhombus	Square
The diagonals are congruent	**No**	**No**	**No**	**Yes**
The diagonals bisect each other	**Yes**	**Yes**	**Yes**	**Yes**
Each diagonal bisects a pair of opposite angles	**No**	**Yes**	**Yes**	**Yes**
The diagonals form 2 pairs of congruent triangles	**Yes**	**Yes**	**Yes**	**Yes**
The diagonals are perpendicular	**Yes**	**Yes**	**Yes**	**Yes**
The diagonals form 4 congruent triangles	**No**	**No**	**No**	**Yes**

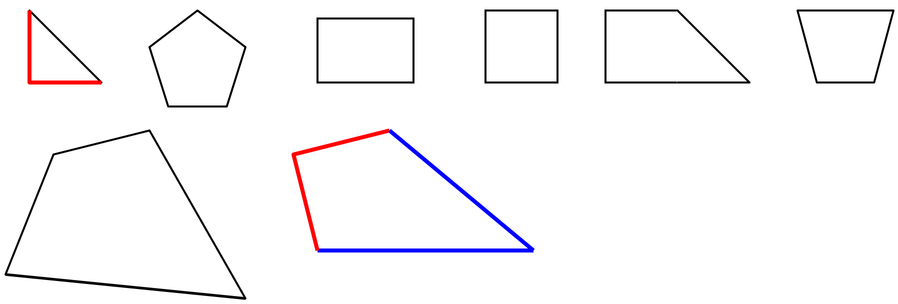
True or false:
1) F 2) T 3) T 4) F 5) F 6) T 7) F 8) F 9) T
10) T 11) T 12) T 13) T 14) T 15) T 16) T
17) T 18) F 19) F 20) T

Solve quadrilateral properties:

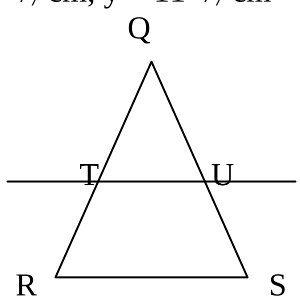
21) ZO = 5cm 22) ZO = 5 cm 23) WO = 4 cm 24) YO = 4 cm
25) Angle WOZ ≡ Angle YOX 26) Angle WOX ≡ YOZ
27) Angle ZWO ≡ XYO 28) Angle ZOY ≡ XOW
29) Angle ZOW ≡ XOY
30) LQPM and MPON; NOP and LQP 31) 90º
32) (a) QO ≡ MS (b) QPM and OPM

Solutions to review of units 4 and 5

1. A polygon is made up of triangles. The number of triangles in a polygon = number of sides – 2; each triangle has 180º. The sum of the interior angles of a triangle = (n-2)180º.
2. There are 18 triangles in a 20-sided polygon and 3240º.
3. …the measure of each interior angle and each exterior angle.
4. 30º. 5) T 6) Supplementary 7) Angle 1 = 55º; 2 = 105º; 3 = 50º; 4 = 130º
8. Angle 2 = angle 1 + 3; angle 2 = 105º and angles 1 and 3 = 55º + 50º.
9. Angle 5 = 65º; 6 = 108º; 7 = 90º; 8 = 90º; 9 = 50º; 10 = 97º
10.

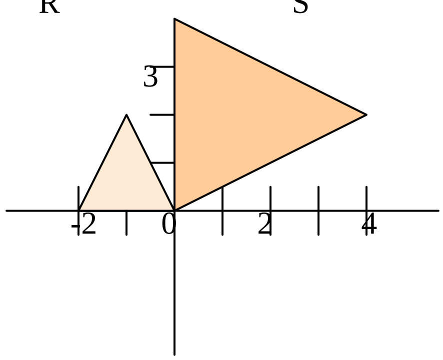


11. (a) isosceles right triangle (b) regular pentagon (c) rectangle (d) square (e) right trapezoid (f) isosceles trapezoid (g) quadrilateral (h) kite
12. Δ ABC ~ DEF by angle-angle-angle; AAA is the reason that figures are similar. Angle A = Angle D; Angle B = Angle E; Angle C = Angle F.
13. The figures are not similar because their corresponding sides do not have equal ratios.
14. Same as # 13.
15. x = $1\,^{1}/3$ feet; y = $1\,^{2}/_{3}$ feet
16. x = $8\,^{2}/_{7}$ cm; y = $11\,^{3}/_{7}$ cm
17.



Angle T = Angle R corresponding angles
Angle U = Angle S corresponding angles
Angle Q = Angle Q same angle
Δ QTU ~ Δ QRS AAA

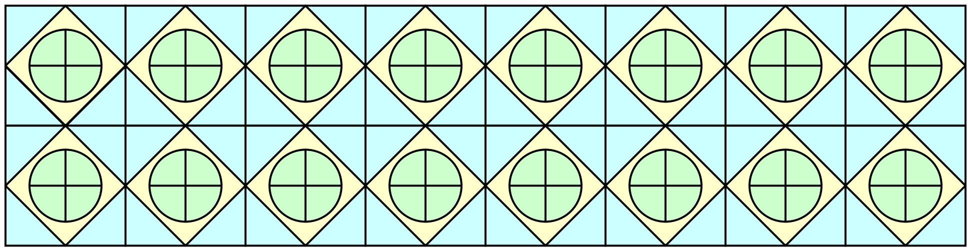
18.



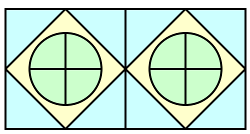
The coordinates of triangle 1:
(-2, 0), (-1, 2), (0, 0)
The coordinates of triangle 2:
(0, 4), (4, 2), (0, 0)
The point of rotation is the same; other points are double the absolute values.

LESSON 6
EVERYDAY GEOMETRY

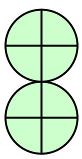
Lesson 6-A
Patterns in Graphic Design



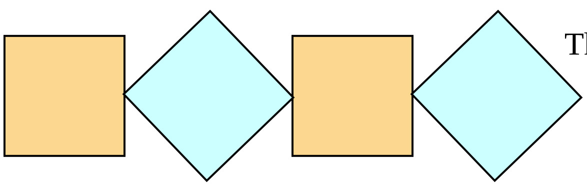
This is a pattern of patio tiles at a seaside resort. It is a tessellation designed from squares. A tessellation is a pattern of tiles without gaps or overlaps. The background squares have a smaller square inside. A circle divided into 4 parts completes the design. The circle design could have been flowers or anything else. The important thing is that central designs are placed in corresponding locations and are congruent.



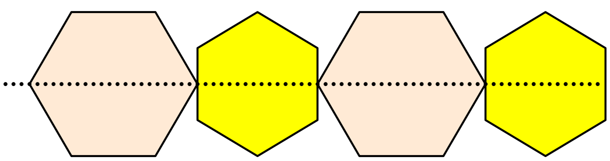
This is a glide or slide translation; the pattern also reflects; it is a mirror image of itself. There is vertical and horizontal line symmetry. The vertical and horizontal lines divide the pattern into 2 perfect halves.



This pattern reflects and has symmetry but does not rotate.



This pattern rotates 45° and glides but does not reflect.



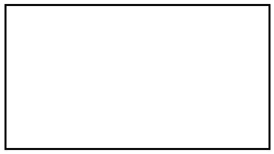
This pattern rotates 90, glides, and dilates. The dotted line indicates horizontal line symmetry.

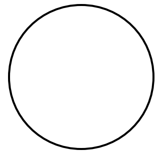
What geometric shapes create the best tessellations? The square, equilateral triangle and the regular hexagon create the best tessellations. Rectangles can be used if they are double the size of the square or if they are inside of another shape. Some will curve like the hexagon and pentagon (soccer ball). M.C. Escher, a Dutch graphic artist made tessellations famous. Check out his work and create designs using his fascinating techniques.

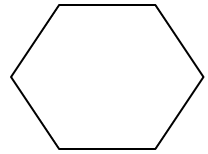
Lesson 6-B
Symmetry in Art and Nature

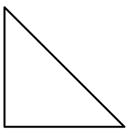
A shape has symmetry when one half is a perfect replica of the other half. There may be **v**ertical, **h**orizontal, or **d**iagonal symmetry. The letters **V**, **H**, and **D** will be used to indicate types of symmetry under each shape.

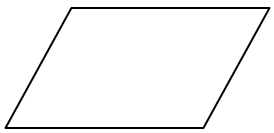

V, H, and D


V and H

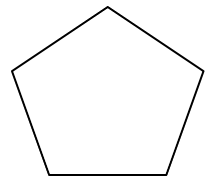

V, H, and D

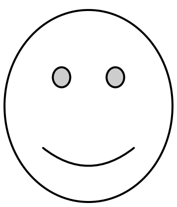

V, H, and D

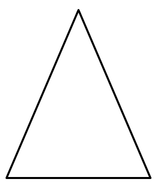

D

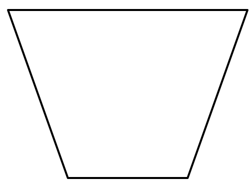

D

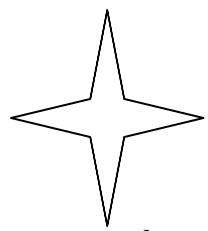

V and H

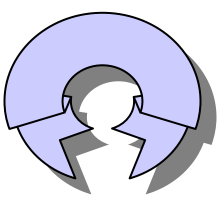

V

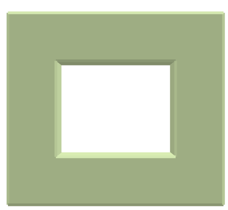

V

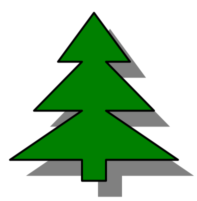

V


V


V and H


V


V, H, and D


V

Study these letters of the alphabet to determine what kind of symmetry each has:

A B C D E F G H I J K L

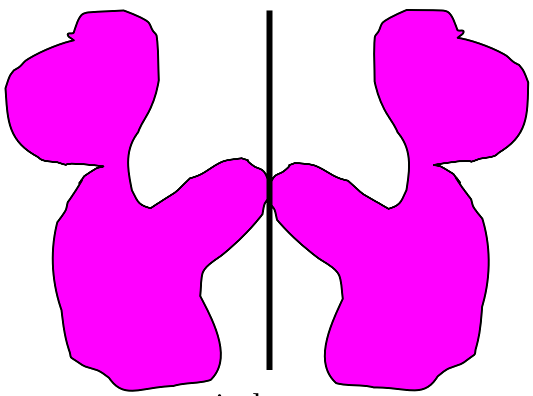
M N O P Q R S T U V W X

Y Z

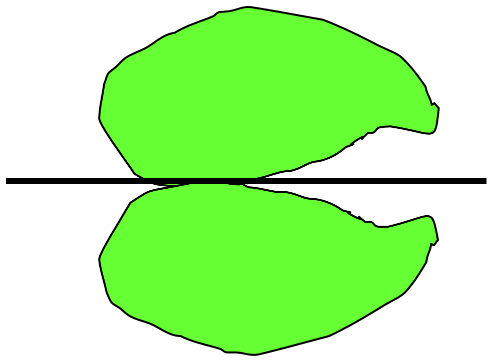
Create your own symmetrical design

A. Materials: Poster ink and paper.

Get a large piece of light colored art paper. Mix a small batch of poster color. Take a large paint brush and drop one or two big blots of paint in the middle of the page. Fold the page down the center vertically, horizontally, or diagonally. Choose only one way as you are new to it. Later you can fold along other symmetrical lines. Open the page and let it dry. You now have your own ink blot symmetry.



vertical symmetry



horizontal symmetry

Lesson 6-C
Mini Project

Create a tessellation using one or any combinations of a square, equilateral triangle, or regular hexagon. Your tessellations should have:

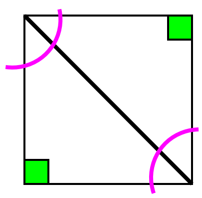
- Reflections
- Rotations
- Symmetry
- Dilations

Use as much color as you like and share it with friends.

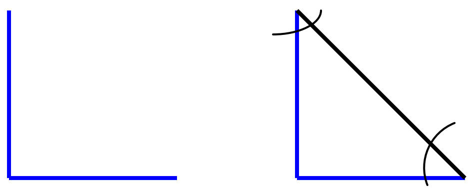
LESSON 7
A HIGHER LEVEL OF GEOMETRY

Lesson 7-A
Constructing Special Right Triangles

The 45º-45º-90º triangle: Start with a square; divide into 2 halves across the diagonal. The diagonal bisects the right angle creating two 45º angles. One right angle remains intact.



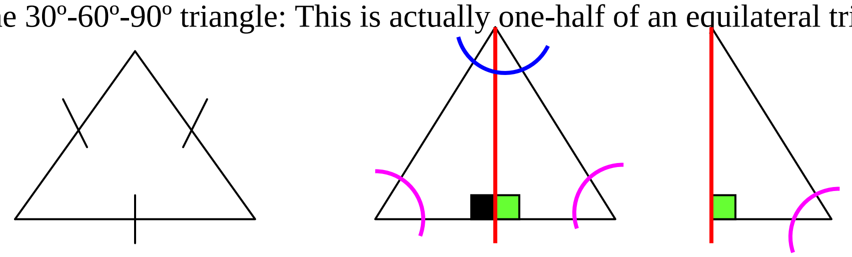
The bold line is the diagonal that splits each right angle into 2 equal parts. The base and one leg are congruent. We have an isosceles right triangle.



Step 1 Step 2

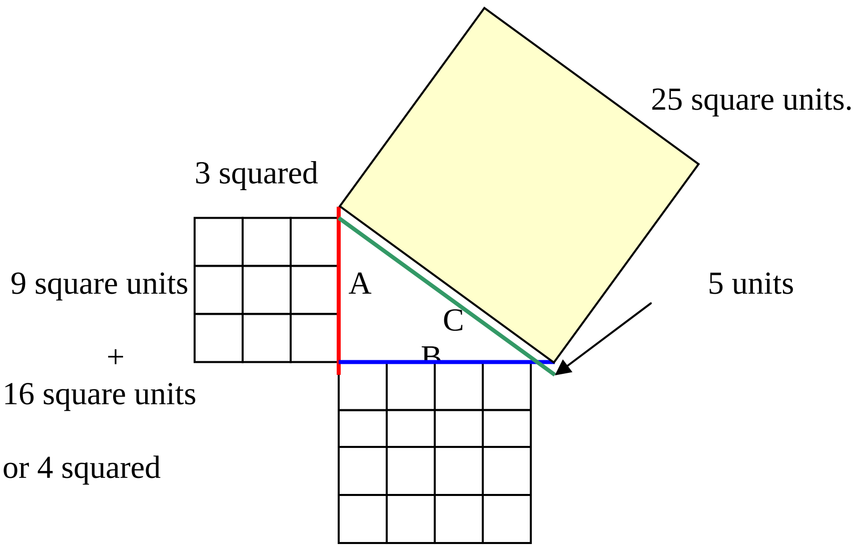
Draw leg A and B as a right angle with congruent sides. Connect the hypotenuse (leg C). Angles opposite congruent sides are congruent; we therefore have an isosceles right triangle.

The 30º-60º-90º triangle: This is actually one-half of an equilateral triangle.



The diagram shows all legs congruent. That means that all angles are also congruent and equal to 60º. When we divide the triangle in half by drawing a median we create 2 right triangles. The angles near the median at the base are 90º. The angles on each side are still 60º. The angle at the vertex is bisected into two 30º angles.

Pythagorean Triples: The first one will be done as an example. Use a ruler or straight edge and compass to construct the others. If Leg A or B are done, simply draw in leg C. It should have the measure expected.



Leg A^2 or 3 squared **+** **Leg B^2** or 16 square units **=** **C squared** or 25 square units

$A^2 + B^2 = C^2$ $C = \sqrt{A^2 + B^2}$

Pythagorean Triangles with Legs that are Whole Numbers

Leg A	Leg B	Leg C or Hypotenuse
3	4	5
5	12	13
7	24	25
8	15	17
9	40	41
11	60	61
13	84	85
12	35	37

For more Pythagorean Triples go to:
http://www2.math.uic.edu/~fields/puzzle/triples.html
http://www.saltire.com/applets/pythag/incircle.html
http://www.math.rutgers.edu/~erowland/tripleslist.html
(All websites Accessed: 1/23/2006)

Try constructing the 5-12-13, or the 8-15-17. Others may have sides that are too long to fit on a regular page.

Many of the Pythagorean Triples have prime numbers. One can enlarge any 3 legs of a Pythagorean Triple to obtain a dilated right triangle. We will learn how to work with right triangles that do not have whole numbers on all legs.

Skills needed for this unit

- How to find squares and square roots of perfect and imperfect squares
- Area formula
- Area of a circle and knowledge of circle parts
- Working with Pi as a decimal and as a fraction; working with Pi without calculating Pi
- Ratios and proportions
- Patterns and sequences
- Inequality symbols
- Factoring

Perfect squares: A perfect square is the product of any integer multiplied by itself.

$1^2 = 1$	$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$	$6^2 = 36$
$7^2 = 49$	$8^2 = 64$	$9^2 = 81$	$10^2 = 100$	$11^2 = 121$	$12^2 = 144$
$13^2 = 169$		$14^2 = 196$	$15^2 = 225$	$16^2 = 256$	$17^2 = 289$
		$18^2 = 324$	$19^2 = 361$	$20^2 = 400$	

The square root of a number answers the question: What number times itself = x?

$4^2 = 16$ so $\sqrt{16} = 4$ $10^2 = 100$, so $\sqrt{100} = 10$, and $\sqrt{144} = 12$

Sometimes we do not calculate certain roots

The root of imperfect squares

Let's start by writing the first 20 perfect squares: (**Students should know these by sight. They are written above.**)

1	4	9	16	25
36	49	64	81	100
121	144	169	196	225
256	289	324	361	400

Example 1

Solve:

(a) $\sqrt{2}$ = > $\sqrt{1} < \sqrt{4}$ = > $1 < 2$ (two sits between two perfect squares, 1 and 4)

(b) $\sqrt{5}$ = > $\sqrt{4} < \sqrt{9}$ = > $2 < 3$

$\sqrt{2}$ and $\sqrt{5}$ are often not calculated.

Example 2

(a) $\sqrt{24} = (\sqrt{4})(\sqrt{6}) = 2\sqrt{6}$ (b) $\sqrt{45} = \sqrt{9}(\sqrt{5}) = 3\sqrt{5}$

(c) ***$\sqrt{38} = \sqrt{2}(\sqrt{19})$*** **DO NOT FACTOR**   (d) ***$\sqrt{42} = \sqrt{6}(\sqrt{7})$*** **DO NOT FACTOR**

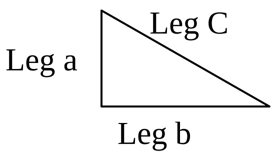
What is the method here? If the number can be factored using a perfect square, do it; if not leave it as it is!

Quick Practice

(a) $\sqrt{25}$	(b) $\sqrt{36}$	(c) $\sqrt{18}$	(d) $\sqrt{5}$
(e) $\sqrt{10}$	(f) $\sqrt{12}$	(g) $\sqrt{20}$	(h) $\sqrt{26}$
(i) $\sqrt{14}$	(j) $\sqrt{50}$	(k) $\sqrt{56}$	(l) $\sqrt{98}$

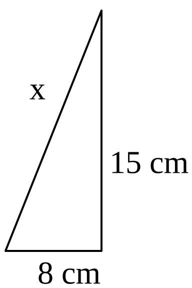
The Pythagorean Theorem

The Pythagorean Theorem is used to determine the **lengths of the sides of right triangles.** The sides of right triangles are called **legs**; leg A or leg B are interchangeable bases. **Leg C is the longest side and is called the hypotenuse.**



We use the formula $a^2 + b^2 = C^2$

$$C = \sqrt{a^2 + b^2}$$



If leg a = 15cm and leg b = 8 cm, find leg C.

Always start with the formula and substitute values.
$a^2 + b^2 = C^2 = 8^2 + 15^2 = C^2 = 64 + 225 = 289$
$C^2 = 289$; $C = \sqrt{289} = 17$ cm. **C = 17 cm**

If we find that the square of leg a + the square of leg b = the square of leg C, then the triangle is a right triangle.

Example 3: If legs a and b of a right triangle are 5 cm and 7 cm, find the measure of leg C. Is this a Pythagorean Triple?
$a^2 + b^2 = C^2 = 5^2 + 7^2 = C^2 = 25 + 49 = 74$; $C^2 = 74$; **$C = \sqrt{74} = 8.6$ cm**; **this is not a Pythagorean Triple because C = 8.6 cm is not a whole number.**

Sometimes you may be asked to leave your answer as a square root.

Example 4: Leg C of a right triangle is 25cm; leg a is 7 cm; what is the measure of leg b? Is it a Pythagorean Triple? Explain.
$a^2 + b^2 = C^2 = 7^2 + x = 25^2 = 49 + x = 625$; $x = 625 – 49$; leg $b^2 = 576$;
leg b = $\sqrt{576} = 24$ cm. **This is a Pythagorean Triple because the square roots are all whole numbers**

Example 4: A triangle measured 10m, 24m, and 26m. Is it a right triangle?
The longest side = leg C = 26m; Let leg a = 10 m and leg b = 24 m
Check to determine whether $10^2 + 24^2 = 26^2 = 100 + 576 = 676$m.
This is a right triangle.

Practice finding square roots online: http://www.aplusmath.com/Flashcards/sqrt.html

Practice

Write T or F after each statement:

1. $\sqrt{95} > 9 < 10$
2. A perfect square has decimals in its root.
3. All squares are perfect.
4. A square root can be negative and positive.
5. $2\sqrt{4} = 4$
6. $\sqrt{60} = 2\sqrt{15}$
7. A Pythagorean triple can be made of imperfect squares.
8. 6, 8, and 10 is a Pythagorean triple.
9. $a^2 + \underline{b^2 = C}$
10. $a = \sqrt{C^2 - b^2}$

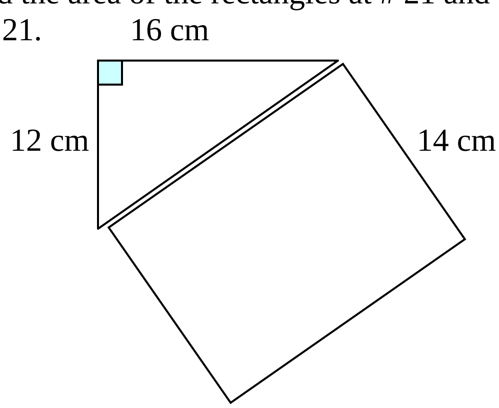
Are they right triangles?

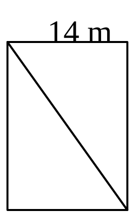
11. 7, 24, 25
12. 3, 4, 5
13. 8, 10, 15
14. 16, 30, 34
15. 20, 21, 29

Find the length of the missing sides:

16. Leg a = 12 m, leg C = 20 m, leg b = __________________
17. Leg C = 85 inches, leg b = 84 inches, leg a = _______________
18. Leg b = 35feet, leg a = 12 feet, leg C = x
19. Leg a = 16 cm, leg b = 63 cm, leg C = __________________
20. Leg b = 15m, leg C = 17 cm, leg a = ___________________

Find the area of the rectangles at # 21 and 22:

21.


22.


The diagonal is 50 m.

Solutions on page 74

Simplifying Radicals

We can add, subtract, multiply, and divide radicals like rational numbers.

Example 1: $\sqrt{9} + \sqrt{25} = 3 + 5 = 8$
Treat each root as if it were parentheses

Example 2: $\sqrt{9 + 25} = \sqrt{34} = > 5 < 6$ or 5.83; compare with example 2 on page 70.

Example 3: $\sqrt{36} - \sqrt{4} = 6 – 2 = 4$

Example 4: $\sqrt{25} \times \sqrt{4} = 5 \times 2 = 10$

Example 5: $3 (\sqrt{16}) = 3 \times 4 = 12$

Example 6: $5 (\sqrt{25}) \times 2 \sqrt{3} = 5 \times 5 \times 2 \sqrt{3} = 50 \sqrt{3}$ sometimes we can leave the answer with the radical. The answer calculated is $50 \times 1.732 = 86.6$

Example 7: (a) $\sqrt{16} \div \sqrt{4} = 4 \div 2 = 2$ (b) $16 \div \sqrt{4} = 16 \div 2 = 8$

Example 8: (a) $(\sqrt{25})^2 = 5^2 = 25$ (b) $- \sqrt{81} (3^2) = - 9 (9) = - 81$

Sometimes irrational numbers are not calculated
$\sqrt{3}, \sqrt{5}, \sqrt{7}, \sqrt{11}$ may be written in their radical form

Practice (Solutions Below)

Simplify each expression:
1. Write as an inequality: (a) $\sqrt{12}$ (b) $\sqrt{48}$ (c) $\sqrt{36}$
2. Use factoring to simplify: (a) $\sqrt{72}$ (b) 20 (c) 90

3. $\sqrt{28} + \sqrt{25}$ 4) $- \sqrt{49} (2)$ 5) $\sqrt{64} \div - \sqrt{4}$ 6) $3 \sqrt{4} + 2 \sqrt{9}$
7. $(\sqrt{169})^2$ 8) $- (\sqrt{81}) + \sqrt{100}$ 9) $[7 (\sqrt{36})]$ 10) $4 \sqrt{10} \cdot 3 \sqrt{10}$

Solutions to page 70, 72, and 73

Quick practice page 70: (a) 5 (b) 6 (c) $\sqrt{9} \cdot \sqrt{2} = 3 \sqrt{2}$ (d) $\sqrt{5}$ (e) $\sqrt{10}$
(f) $\sqrt{4} \cdot \sqrt{3} = 2 \sqrt{3}$ (g) $\sqrt{4} \cdot \sqrt{5} = 2 \sqrt{5}$ (h) $\sqrt{26}$ (i) $\sqrt{14}$
(j) $\sqrt{25} \cdot \sqrt{2} = 5 \sqrt{2}$ (k) $\sqrt{4} \cdot \sqrt{14} = 2 \sqrt{14}$ (l) $\sqrt{49} \cdot \sqrt{2} = 7 \sqrt{2}$

Practice page 72: 1. T 2) F 3) F 4) T 5) T 6) T 7) F 8) T 9) F 10) T

Are they right triangles? 11) Yes 12) Yes 13) No 14) Yes 15) Yes

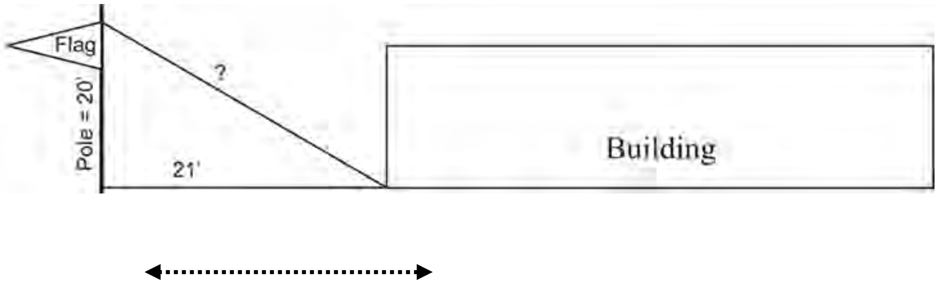
Find the length of the missing sides: 16) 16cm 17) 13 inches 18) 37 feet
19) 65 cm 20) 8cm

Find the area of the rectangles: 21) 280 cm^2 22) 672 m^2

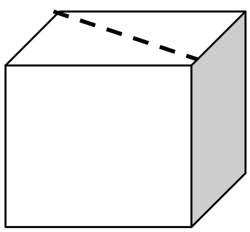
Practice page 73: 1 (a) $> 3 < 4$ (b) $> 6 < 7$ (c) 6 2 (a) $\sqrt{8} \cdot \sqrt{9} = 3 \sqrt{8}$
(b) $\sqrt{4} \cdot \sqrt{5} = 2 \sqrt{5}$ (c) $\sqrt{9} \cdot \sqrt{10} = 3 \sqrt{10}$ 3) $\sqrt{53}$ 4) – 14 5) $8 \div - 2 = -4$
6) $3 \cdot 2 + 2 \cdot 3 = 36$ 7) 169 8) $- \sqrt{81} + \sqrt{100} = - 9 + 10 = 1$
9) $7 \sqrt{36} = 7 \times 6 = 42$ 10) $4 \times 3 (\sqrt{10}) = 12 \sqrt{10}$

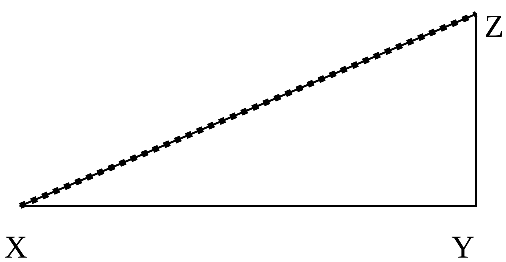
Real-world applications of the Pythagorean Theorem

1. In a cross-country race an athlete runs 30 meters east then 40 meters north. How far is he from the starting point? Draw if necessary.
2.



A flag pole 20 feet tall is to be erected 21 feet from a building. How long should the anchor cable be?

3. The foot of a 6m ladder is 2.5 m away from a wall. How high up the wall does the ladder reach?
4. Find the length of a diagonal of a square that is 15 feet by 15 feet.
5.  This box measures 36 inches by 20 inches across the top. Find the length of the diagonal.
6.



ZY is a street light 5.5 feet tall. A shadow XY is cast along the ground, 7.3 feet long. How far away is the top of the street light from the end of its shadow?

7. A young girl 150 cm tall cast a shadow that was 200 cm long. Her father's shadow was 240 cm long. How tall was her father? (Clue: Think similar triangles)

Solutions to page 72

T or F: 1) T; 2) F; 3) F; 4) T; 5) T; 6) T; 7) F; 8) T; 9) F; 10) T

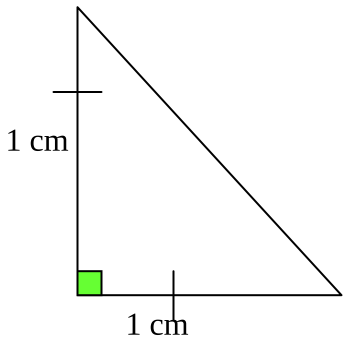
Are they right triangles?: 11) Yes; 12) Yes; 13) No; 14) Yes; 15) Yes

Length of missing sides: 16) 4 m; 17) 13 inches; 18) 37 feet; 19) 65 cm; 20) 8 cm

Area of 21 and 22: Use Pythagorean Theorem to find the length of the diagonal at #21. Leg C = 20; A = 20 x 14 = 280 cm^2
Find leg B at #22. Leg B = 48; A = 14 x 48 = 672 m^2

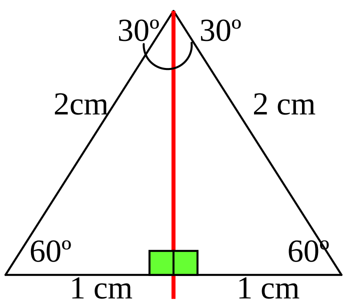
Formulas and Patterns for Special Right Triangles

The isosceles right triangle also called the 45º-45º-90º or the 45 right triangle



Find the measure of the hypotenuse. Start with the Pythagorean Theorem.
$a^2 + b^2$
$C^2 = 1^2 + 1^2$
$C^2 = 1 + 1$
$2 = C^2$
$\sqrt{2} = C$
Because each side is congruent, one can consider each side to be leg a.
The pattern for 45-right triangles is: a, a, a$\sqrt{2}$ where leg C = a$\sqrt{2}$

The 30º-60º-90º triangle or 30º-60º-right triangle

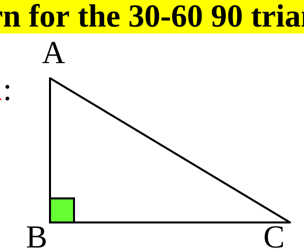


The 30-60-right triangle is one-half of an equilateral triangle. The median divides the equilateral into 2 congruent triangles. After the division the measures of each triangle are leg C = 2 cm, leg b = 1 cm, and the height or leg a (x). What is x? Let's discover. Start with the Pythagorean Theorem.

$a^2 + b^2 = C^2$
$x^2 + 1^2 = 2^2$
$x + 1 – 1 = 4 - 1$
$x = 3$
Let's call each leg "a". One leg is a, another is 2a, the last leg is a$\sqrt{3}$

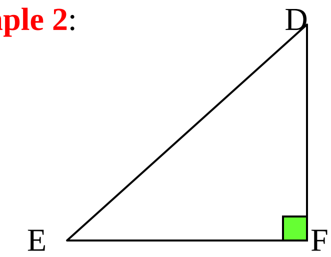
The pattern for the 30-60 90 triangle is: a, 2a, a$\sqrt{3}$ where 2a is leg C, the hypotenuse.

Example 1:



A = 60º; C = 30º; AB = 4cm; use the pattern to find the measure of BC and AC.
Let AB = leg a. The pattern is a, 2a, a$\sqrt{3}$
A = 4cm; answer = 4cm, 8cm, 4$\sqrt{3}$
BC = 4$\sqrt{3}$ AC = 8 cm

Example 2:

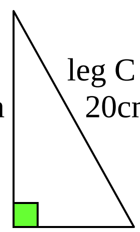


Angle D = 45º; angle E = 45º; DF = 7ft. Find DE and EF. Use the pattern a, a, a√2
(since we have an isosceles right triangle)
Leg a = 7 ft; leg b = 7 ft; leg C = 7√2 ft

Example 3: The hypotenuse of a 30-60-right triangle is 12 feet. What are the measures of legs a and b?

If leg C = 12 feet or 2a, then leg a = 6 feet (one half of C) and leg b = 6 √3 feet

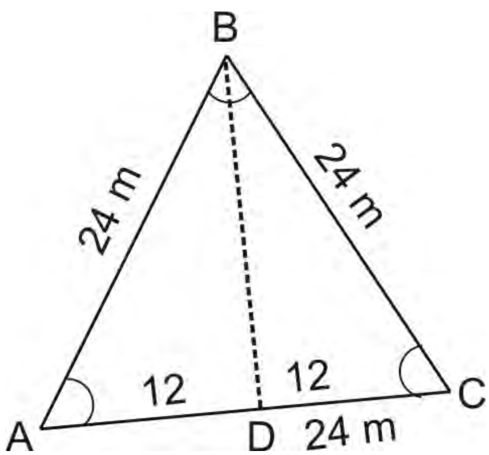
Example 4: Find the area and perimeter of a 30-60 right triangle with a height (leg a) of 10 cm.
First find the measures of the base or leg b and leg C, the hypotenuse.
Leg a = 10 cm; leg b = 10√3 cm; leg C = 20cm
Draw a triangle and place the measures in order to better visualize the results.



Area of the triangle = ½ (bh) = ½ (10 √3) 10
= 5 · 10 · √3 = 50 √3
Simplify further or leave the answer as is. 50 · 1.732
= **$86.8cm^2$ or 50 √3 cm^2.**
Perimeter of the triangle = 10cm + 20 cm + 10 √3 cm
= 30 √3;
Simplify or leave as is: **51.96 cm or 30 √3 cm**

Knowledge of the Pythagorean Theorem and the properties of special right triangles allow us to find the measure of all sides of a right triangle from just one measure.

Example 5: This **equilateral triangle is 24 m on each side. Find its area.**



First divide the triangle into 2 equal parts. Use those measures to find the height. Apply the formula for area of a triangle to solve**. First, find the height BD**.

Each 30-60-right triangle has a base of 12 m (leg b) and a hypotenuse of 24m (leg C). The pattern is a, 2a, a√3. The height is 12 √3; the area is ½ (12 · 12 √3). This is only half of the triangle; multiply by 2 to get the area of the whole triangle.
½ (12 · 12 √3 · 2) = 144√3 m^2 or 249.41 m^2

Find BD:
$a^2 + b^2 = c^2$
$(12^2) + b^2 = 24^2$
$B^2 = 24^2 - 12^2$
$B^2 = 432$
$B = \sqrt{432}$
$B = 12\sqrt{3}$ is the height BD

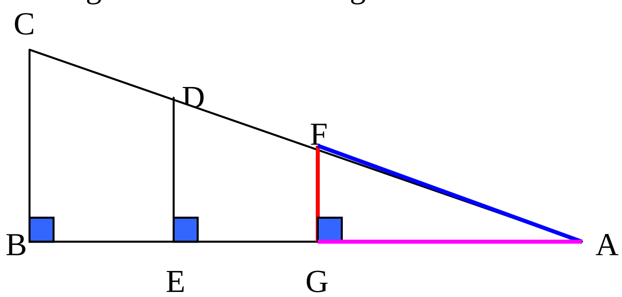
Find Area of ABD (1/2 the triangle ABC):

$$A = \frac{1}{2}(12 \text{ x } 12\sqrt{3}) = \frac{12 \times 12\sqrt{3}}{2} = 6 \times 12\sqrt{3} = 72\sqrt{3}$$

$$2 \times 72\sqrt{3} = 144\sqrt{3}^{\,2}$$

Trigonometric Ratios

Trigonometry means triangle measure. Trigonometric ratios are like the ratios of similar triangles. Study the diagram of similar triangles:



There are 3 similar triangles in the diagram; Δ ABC ~ AED ~ AGF. Angle A is common to all triangles; CB, DE, and FG are all parallel, so the corresponding right angles are congruent; and corresponding angles BCA, EDA, and GFA are congruent (AAA). Their ratios must be equal. **Let's use angle A as the anchor**.

The sine of an angle in a right triangle equals the length of the opposite side divided by the length of the hypotenuse.

The ratio of the $\dfrac{Length\_of\_the\_side\_opposite\_angle\_A}{Length\_of\_the\_hypotenuse} = \dfrac{a}{c}$ is called the sine of angle A or sin A

The cosine of an angle in a right triangle equals the adjacent side divided by the hypotenuse.

The ratio of the $\dfrac{Length\_of\_the\_side\_adjacent\_angle\_A}{Length\_of\_the\_hypotenuse} = \dfrac{b}{c}$ is called the cosine of angle A or cos A

The tangent of an angle in a right triangle equals the opposite side divided by the adjacent side.

The ratio of the $\dfrac{Length\_of\_the\_side\_opposite\_angle\_A}{Length\_of\_the\_adjacent\_side} = \dfrac{a}{b}$ is called the tangent of angle A or tan A

NOTE: There are three other trigonometric ratios not covered. They are cotangent (cot), secant (sec), and cosecant (csc);

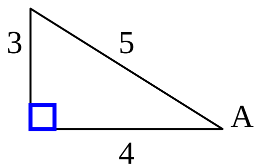
cot A = $\dfrac{\cos\_A}{\sin\_A} = \dfrac{adj}{opp}$ sec A = $\dfrac{1}{\cos\_A} = \dfrac{hyp}{adj}$ csc A = $\dfrac{1}{\sin\_A} = \dfrac{hyp}{opp}$

Right triangle ratios help us to find the measure of the angles.
The ratios depend on the measure of angle A not on the size of the right triangle.

The value of a trigonometric ratio depends only on an acute angle (<90◦).

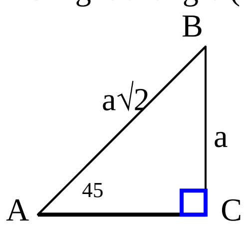
A helpful tool to aid memory is the mnemonic: **S**OH-**C**AH-**T**OA.
S-**sin**: opposite/hypotenuse; C-**cos**: adjacent/hypotenuse; T-**tan**: opposite/adjacent

Example 1: Find the sine, cosine and tangent of angle A



Sin A = 3/5 or .6; Cos A = 4/5; Tan A = ¾

Example 2: Find sin 45º, cos 45º, and tan 45º. Since we are working with right triangles and angle A is 45º, then the triangle must be the 45-right. Think of the ratio/pattern for the 45-right triangle (a, a, a√2). **Draw it.**



Sin 45º = $\frac{a}{a\sqrt{2}} = \frac{a\sqrt{2}}{a\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}}{2}$; Cos 45º = same as sin 45º

Tan 45º = $\frac{a}{a}$ = 1

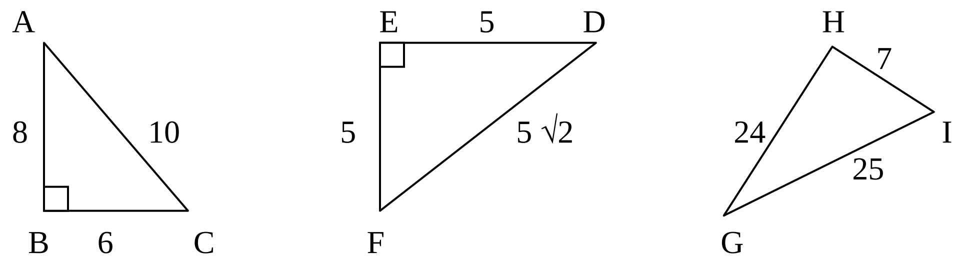
Example 3: Simplifying with radicals

(a) $\dfrac{a}{a\sqrt{3}}$ → remove the radical from below the line; multiply numerator and denominator by the radical

$= \dfrac{a \cdot \sqrt{3}}{a \cdot \sqrt{3} \cdot \sqrt{3}} = \dfrac{\sqrt{3}}{3}$; the a's divide out; squaring the root eliminates the radical sign.

(b) $\dfrac{6}{4\sqrt{2}} = \dfrac{6 \cdot \sqrt{2}}{4 \cdot \sqrt{2} \cdot \sqrt{2}} = \dfrac{3\sqrt{2}}{2 \cdot 2} = \dfrac{3\sqrt{2}}{4}$ or ¾ $\sqrt{2}$

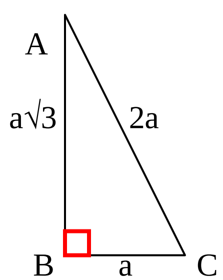
Practice



Express the ratio of each right triangle as a fraction in simplest form:

1. Sin A	2) Cos A	3) Tan A	4) Sin C
5. Cos C	6) Tan C	7) Sin F	8) Cos F
9. Tan F	10) Sin D	11) Cos D	12) Tan D
13. Sin G	14) Cos G	15) Tan G	16) Sin I
17. Cos I	18) Tan I		

Example 4: Find the sin, cos, and tan of a 30º-60º-90º triangle



The measure of angle A is 30º; angle C is 60º. Find the sin, cos, and tan, of each of those angles.
The common terms will be factored in each ratio.

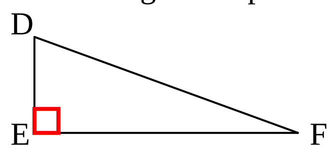
Sin 30º $= \dfrac{a}{2a} = \dfrac{1}{2}$; **cos 30º** $= \dfrac{a\sqrt{3}}{2a} = \dfrac{\sqrt{3}}{2}$; **tan 30º** $= \dfrac{a}{a\sqrt{3}} = \dfrac{a \cdot \sqrt{3}}{a \cdot \sqrt{3} \cdot \sqrt{3}} = \dfrac{\sqrt{3}}{3}$

Sin 60º $= \dfrac{a\sqrt{3}}{2a} = \dfrac{\sqrt{3}}{2}$; **cos 60º** $= \dfrac{a}{2a} = \dfrac{1}{2}$; **tan 60º** $= \dfrac{a\sqrt{3}}{a} = \sqrt{3}$

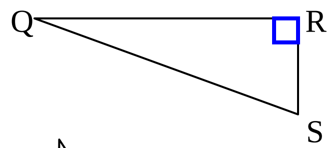
The sin of 30º is the cos of 60º; the sin of 60º is the cos of 30º; the tan of 30º is the reciprocal of the tan of 60º. Be sure to express each answer in simplest radical form.

Try these

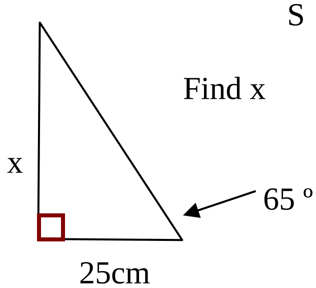
1. Use trig tables to determine the smallest value for sine.
2. What is the greatest possible value for sine.
3. If D = 67º , the sin D = .9205, and DF = 7m, what is the measure of EF and DE?



4. Angle S = 65º; cos 65 = .4226, QS = 9 feet; find the measure of QR and RS.



5. Find x



6. Trigonometric tables provide the numerical value of each ratio. Follow the links:

http://www.math.com/tables/trig/tables.htm
http://www.sosmath.com/tables/trigtable/trigtable.html
http://www.brassmein.com/tech/tech001.htm
(All websites Accessed: 1/24/2006)

A scientific calculator such as the Texas Instruments 30XA will work as well

7. Use trig tables to find the measure of each angle to the nearest degree:

(a) sin A = .0698 (b) tan A = .2126 (c) cos A = .7880
(d) tan B = .5774 (e) sin B = .7071 (f) cos B = .6428

8. Use trig tables or a scientific calculator to find the value of each to four decimal places:

(a) cos 30º – sin 30º (b) cos 30º + cos 10º (c) sin 90º + cos 30º
(d) tan 45º + tan 0º (e) 2 cos 45º – cos 90º (f) cos 60º – 2 cos 30º

Solutions to pages 80 and 81

Page 80:

1. 3/5	2) 4/5	3) ¾	4) 4/5	5) 3/5	6) 4/5
7. $\frac{\sqrt{2}}{2}$	8) $\frac{\sqrt{2}}{2}$	9) 1	10) $\frac{\sqrt{2}}{2}$	11) $\frac{\sqrt{2}}{2}$	12) 1
13. $\frac{7}{25}$	14) $\frac{24}{25}$	15) $\frac{7}{24}$	16) $\frac{24}{25}$	17) $\frac{7}{25}$	18) $\frac{24}{7}$

Page 81:

1. Smallest possible value for sine = 0 2) Largest possible value for sine = 1

3. EF = 6.4 cm. **Explanation**: The sin of D is a ratio of **opposite over hypotenuse;** substitute the values given and solve for EF (x). $\frac{EF \text{ or } x}{7} = .9205$

EF = .9205 x 7 = 6.4 m

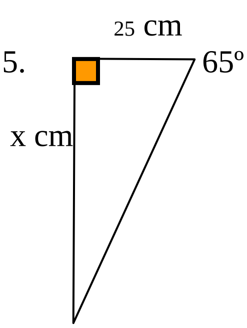
Now you have 2 sides of a right triangle, 7m on the hypotenuse and 6.4 m on leg A. Use the Pythagorean Theorem to solve for leg B. $C^2 = A^2 + B^2$

$7^2 = 6.4^2 + x^2$ (x = DE) = $49 + 40.94 + x^2$; $x^2 = 8.04$; $x = \sqrt{8.04} = 2.8$; DE = 2.8 cm

4. **QR = 8.158 and RS = 3.8**. **Explanation**: The cosine of S or 65º is the ratio of the adjacent side divided by the hypotenuse. The cos 65 = .4226; using the same reasoning as for #3, $\frac{x}{9} = .4226$; x = .4226 (9) = 3.8 or RS.

Apply the Pythagorean Theorem to solve for QR.

5.



The relationship closest to 65º is the **opposite side** and the **adjacent side**. That indicates that we must use the tan of 65º. Use trig tables to find the numerical value of that ratio (2.145). Use proportion to find the missing value: Opposite over adjacent or $\frac{x}{25} = 2.145$; x = 2.145(25)

x = 53.625 cm

7. (a) 4º (b) 12º (c) 38º (d) 30º (e) 45º (f) 40º

Explanation: Use trig tables, look in the sine, cosine, or tan column for the value closest to the one given, and read the degree equivalent. That's it!

8. (a) .3660 (b) 1.8508 (c) 1.8660 (d) 1 (e) 1.4142 (f) -1.232

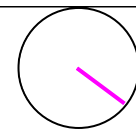
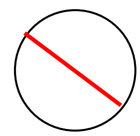
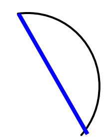
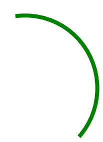
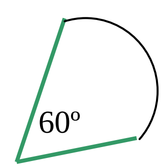
Explanation: Use trig tables or a scientific calculator to find the numerical value of each ratio, and then calculate. (a) Cos 30º – sin 30º = .8660 - .5000 = .3660

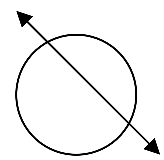
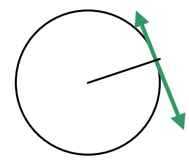
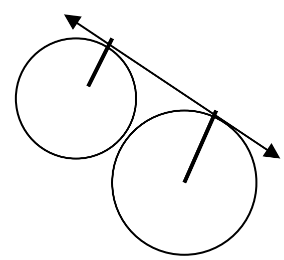
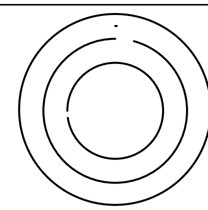
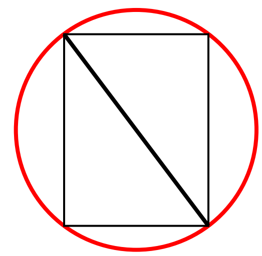
(e) 2 cos 45º - cos 90º = 2(.7071) – 0 = 1.4142

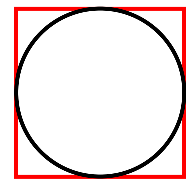
(f) Cos 60º is not the same as 2 cos 30º

Lesson 7-B
Circles and Circle Parts

Review

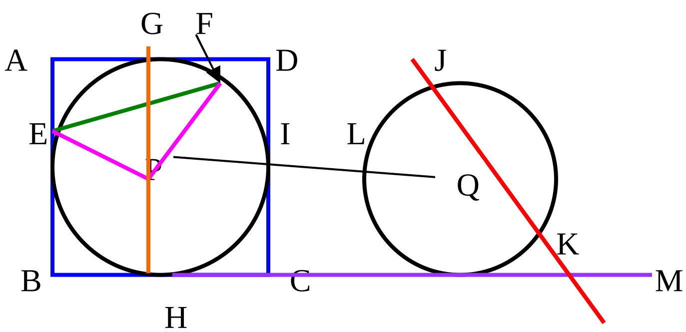
Name of circle part	Definition	Formula or relationship to another part	Example
Center	The point that is equidistant from every point on the circumference	0 degrees; the origin of the circle	•
Radius	The distance between the center and any point on the circumference. All radii in the same circle are congruent.	One half the diameter	
Diameter	The distance across the circle through the center	Two times the radius; a little less than one third of the circumference	
Chord	Any line segment ***originating*** and ***terminating on the circumference,*** while passing through the circle. The diameter is a chord. A chord joins the end-points of an arc.	A chord could be the side of an inscribed polygon.	
Arc	A fraction of the circumference measured in degrees or linear measure.	An arc has the same degree measure as the central angle it defines.	
Pi	An irrational value that defines the ratio of the circumference to the diameter or c/d	Decimal value = 3.14 Fractional value = 22/7 The diameter fits the circumference 3.14 times.	The circumference of a circle was 15.7 cm. The diameter was 5 cm. c/d or 15.7 ÷ 5 = 3.14.
Sector	A part of the circle that is bound by 2 radii; a fraction of the circle. A sector with a central angle of 90º is ¼ of the circle. If the central angle is 60º the sector is ${}^{1}/_{6}$ of the circle.	The measure of the central angle determines the fraction of the circle.	

Name of circle part	Definition	Formula or relationship to another part	Example
Secant of a circle	A line segment that intersects the circle at two points. ***The line segment originates and terminates outside the circle***. The secant contains a chord.	Compare the secant to the chord.	
Tangent of a circle	A tangent is similar to a secant except that it intersects/touches the circle at one point only.	The tangent is perpendicular to the radius at the point where it touches the circle.	
Common tangents	These are tangents that touch 2 or more circles at the same time	A bicycle is made with common tangents; each tangent is perpendicular to the radii of the circles they intersect.	
Concentric circles	Concentric circles have the same center but different radii	They have a relationship of size that is evident in their radii or their diameters.	
Circumscribed circle	A circle that is drawn outside a polygon is circumscribed around that figure. The word **circumscribed means written around.**	The radius and diameter of the circumscribed circle bear a relationship with the polygon around which they are drawn. The diagonal of this rectangle is the diameter of the circle.	

Name of circle part	Definition	Formula or relationship to another part	Example
Inscribed circle	A circle that is drawn inside a polygon is inscribed in that polygon. **The word inscribed means written in or recorded.**	The sides of the square around the circle form the diameter of the circle. Each point of the circle that touches the square is a point of tangency.	

Identify circle parts

Name each part identified by letters in the diagram. Some parts may be written more than once.



AD = ____________________ EF = ___________________ GH = __________________

LJK = ____________________ PI = ____________________ LQ = __________________

JK = ____________________ DC = ___________________ LJ = ___________________

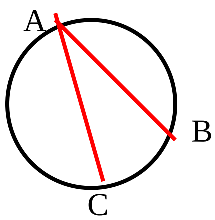
JQK = __________________ EPF = ___________________ HM = _________________

Solutions on page 91

Arcs and Angles

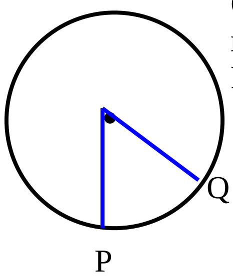
An arc defines a central or an inscribed angle. There are major and minor arcs. As the words suggest, major arcs are bigger and minor arcs are smaller. Three letters will identify major arcs, while two letters define minor arcs.

Examples: AB is a minor arc but ABC is a major arc. Use small arcs over the letters to identify arcs so as not to be confused with line segments or angles.



$\overset{\frown}{AB}$ $\overset{\frown}{ABC}$

There are several arcs identified in the diagram: AC, ACB (same as BCA), AB, BC, and ABC.
AC, BC, and AB are minor arcs (two letters). The others are major arcs (three letters).
CAB is an inscribed angle. The inscribed angle is ½ the measure of the arc or the central angle.

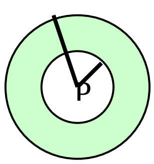


O is the center of this circle. **POQ is a central angle**. The measure of the central angle is also the measure of the arc PQ. The area identified is a sector.

The Area of Circles and Circle Parts

The formula for area of a circle is πr^2. Do not calculate pi, simply write the answer in terms of pi. **Example 1**: Find the area of a circle whose radius is 24 cm.
$A = 24 \cdot 24 \cdot \pi = \mathbf{576\pi\ cm^2}$

Example 2: Find the area of a sector with a central angle of 60º and a radius of 12m.
Area of sector $= \frac{60}{360} \cdot 12 \cdot 12 \cdot \pi = \mathbf{24\pi\ m^2}$
Example 3: Find the area of the shaded part.



The radius of the smaller circle is 7 cm; the radius of the larger circle is 10 cm. The area of the shaded part is the difference in area of the smaller circle and the larger circle.
$(10^2\ \pi) – (7^2\ \pi) = 100\ \pi – 49\ \pi = 51\ \pi\ cm^2$

See page 7 for some basic formulae

Activities

Complete the blanks; write answers in terms of Pi:

Radius	Diameter	Circumference	Area of circle	Central angle	Arc measure in degrees and cm	Area of sector
10 cm				60º		
	24 m				120º	
		5 π feet		100º		
			9 π ins^2		30º	
	15 cm					11.25 cm^2

Complete: The central angle is ___________ times the inscribed angle.

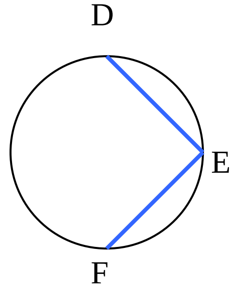
Use your knowledge of the relationship between central and inscribed angles to solve:

Central angle	Inscribed angle
150º	
	32º
	45º
270º	
200º	
	87º
20º	

Important Properties
Of Arcs and Chords

(a) Congruent arcs produce congruent chords.
(b) If a diameter is perpendicular to a chord, it bisects the chord and its arc.
(c) The perpendicular bisector of a chord intersects at the center of the circle.
(d) The perpendicular bisector of a chord helps us to find the radius of the circle.
(e) We can use the Pythagorean Theorem to find the lengths of triangles inscribed in a circle, or the area of triangles inscribed in the circle.

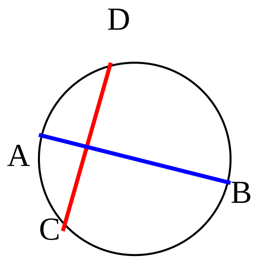
(a)



$\overset{\frown}{DE}$ and $\overset{\frown}{FE}$ are congruent arcs. $\overline{DE}$ and $\overline{FE}$ are congruent chords.

Solutions on page 91

(b)



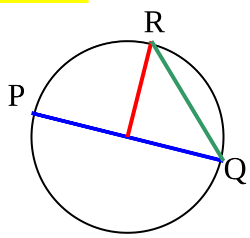
AB is a diameter; CD is a chord. AB bisects chord CD and arc CD. Arc AC ≡ Arc DC

$\overset{\frown}{AC} \equiv \overset{\frown}{DC}$

(c) AB is a diameter; it is perpendicular to chord CD and bisects it. It is therefore a perpendicular bisector that intersects the center of the circle. All diameters pass through the center of a circle.

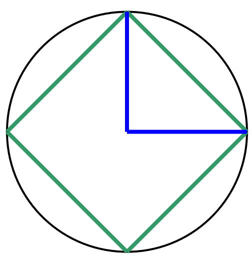
(d) If the perpendicular bisector is a diameter we can use it to find the radius of the same circle.

(e)



O is the center of this circle. ROQ is a central angle of 90º. Arc RQ is 90º (The central angle is congruent to the arc measure). OR and OQ are radii. Knowledge of the radius allows us to find the area of the triangle inscribed in the circle and the sector defined by the radii.

Use a Circle to Measure Central Angles
And the Sides of Regular Polygons



A square is inscribed in the circle.
(a) What is the measure of each central angle and each arc?
(b) The radius is 7 cm. What is the area of the sector identified?
(c) Find the perimeter and area of the square.
(d) Answer in terms of Pi.

Clues you can use

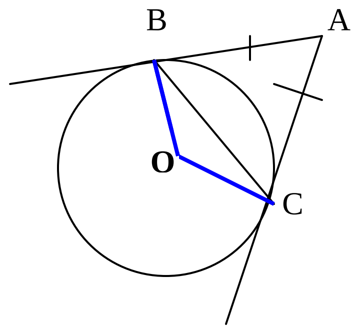
The square is a regular polygon. What is the sum of the interior angles? **360º**.
The central angle is ¼ of the circle; the intercepted arc is congruent to the central angle; therefore each arc is **90º**.

The area of the circle is πr^2. The area of the sector is ¼ the area of the circle.

The sector is a 45º-45º-90º triangle. What is the measure of the hypotenuse?

The length of the hypotenuse is the side of the square. Use that measure to find area and perimeter.

Tangents



AB and AC are tangents to the circle drawn from an exterior point, A. Points B and C are points of tangency. Any radius of the circle drawn to the point of tangency is perpendicular to the tangent. Line segments tangent to a circle from an exterior point are congruent.

$$AB \equiv AC$$

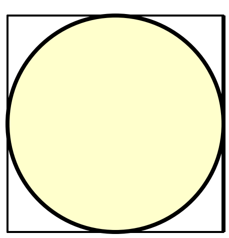
Study the diagram and respond to the questions:

1. Name the quadrilateral ABOC.
2. What is the measure of angles OBA and OCA?
3. If angle A is 60º, what is the measure of angle BOC?
4. The radius of the circle is 5 cm. Find the area of the sector BOC.
5. What is the degree measure of arc BC?
6. What is the linear measure of the arc BC?

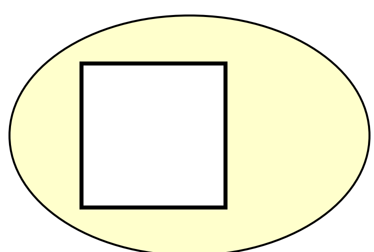
Clues you can use

Review types of quadrilaterals.
The central angle O is congruent to the intercepted arc.
Line segments AC and AB are drawn from exterior points and are tangent to the circle.
Radii OB and OC are congruent.

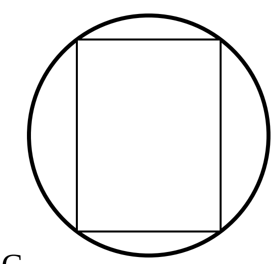
Inscribed, Circumscribed or Neither?
Solutions on page 92



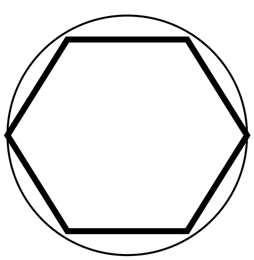
A ____________________



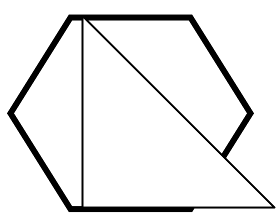
B ________________________



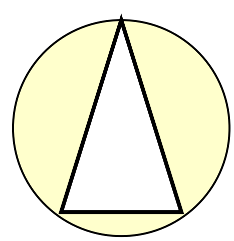
C _____________________



D ____________________



E _______________________



F ______________________

Problems with Tangents

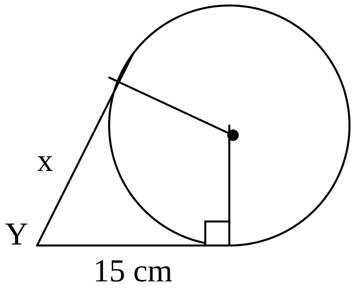


Figure A

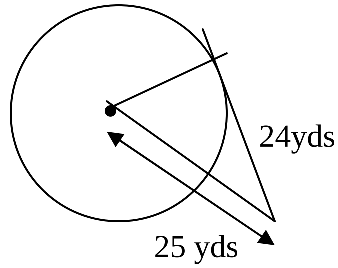


Figure B

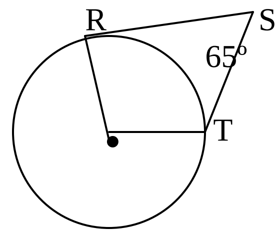


Figure C

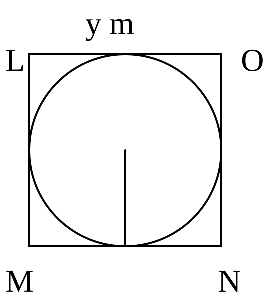


Figure D

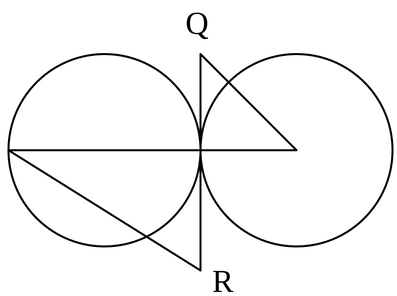


Figure E

1. Y is an external point from which a tangent is drawn. Find x and state the theorem on which your reasoning is based.
2. Is the triangle inscribed in figure B? Explain.
3. Is the triangle in figure B a right triangle? Show your calculations to prove or disprove.
4. Name the quadrilateral at figure C. Point P is the center. If angle S is 65º, what is the measure of the central angle? What is the measure of the intercepted arc?
5. The radius of figure D is 21 inches. Find the area of the circumscribed polygon. Use Pi = 22/7 if necessary.
6. What would be the measure of the diagonal LN in figure D?
7. Figure E is made up of 2 ***congruent*** circles with a common external tangent. If the circumference of one of the circles is 20 π feet, find its radius and area.
8. An isosceles triangle is formed at figure E. What is the length of the hypotenuse?
9. The larger triangle is a 30º-60º right. What is the measure of the hypotenuse?
10. Complete: QR is a ______________. The angles formed at the point of tangency equal __________________.

Solutions to pages 85 and 87

Identify circle parts:
AD = tangent (diameter also); EF = chord; GH = diameter; LJK = major arc; PI = radius; LQ = radius; JK = secant; Dc = tangent; LJ minor arc; JQK = central angle; EPF = central angle; HM = tangent; AD is the side of a circumscribed square.

Complete the blanks; write answers in terms of Pi:

Radius	Diameter	Circumference	Area of circle	Central angle	Arc measure in degrees and cm	Area of sector
10 cm	**20 cm**	**20 π cm**	**100 π cm^2**	60º	**60º; 3.3 π cm**	**16.6 π cm^2**
12m	24 m	**24 π m**	**144 π m^2**	**120º**	120º; **8 π**	**48 π m^2**
2.5 feet	**5 feet**	5 π feet	**6.25 πft^2**	100º	**100º; 1.38 π ft**	**1.73 π ft^2**
3 ins	**6 ins**	**6 π ft**	9 π ins^2	**30º**	30º; **½ π ins**	**¾ π ins^2**
7.5 cm	15 cm	**15 π cm**	**56.25 π cm^2**	**72º**	**72º; 3π cm**	11.25 cm^2

The central angle is ***2 times*** the inscribed angle.

Use your knowledge of the relationship between central and inscribed angles to solve:

Central angle	Inscribed angle
150º	**75º**
64º	32º
90º	45º
270º	**135º**
200º	**100º**
174º	87º
20º	**10º**

Page 88: (a) 90º (b) Area of the sector = 49/4 π cm^2; (c) Perimeter of the square = 28 √2 cm; explanation: one side of the square = the hypotenuse of the triangle; the triangle is an isosceles right with a pattern of a, a, a√2 for the measure of sides. A = 7 cm (radius), so the hypotenuse = 7√2 cm and p = 4 · 7 √2. Area of square = 49 π cm^2; (d) All answers are in terms of Pi.

Page 89: 1) ABOC is a kite; 2) OBA and OCA = 90º; 3) angle BOC = 120º; 4) Area of sector = 25/3 π cm^2; 5) Arc BC = 120º; 6) Linear measure of arc BC = 1/3 of the circumference = 10/3 π cm^2.

Page 89, inscribed, circumscribed, or neither?

A = circumscribed square, inscribed circle
B = Neither
C = Inscribed rectangle, circumscribed circle
D = Inscribed hexagon; circumscribed circle
E = Neither
F = Inscribed triangle; circumscribed circle

Real-world Applications and Problem Solving

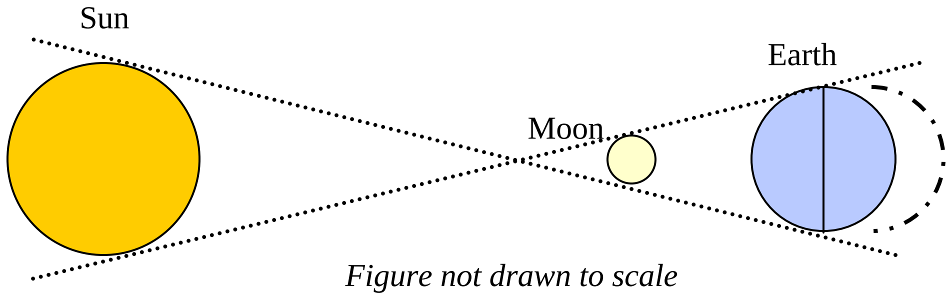


Figure not drawn to scale

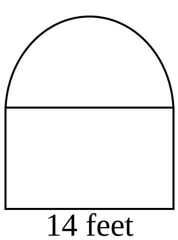
1. Imagine that this represents an eclipse of the sun in which the moon blocks the sunlight from reaching the earth. One half of the earth will experience its normal night and the other half will see the solar eclipse (dark).

(a) What geometric term would be given to the outer edges of the shadow (dotted lines) the moon casts?
(b) Name all the types of angles formed by this configuration.
(c) What geometric term is given to the line dividing one half of the earth from the other half?
(d) The point on the earth where the outer edge of the shadows touch the earth would be

____

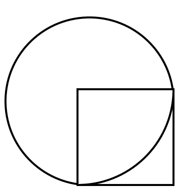
(e) The moon's shadow would fall on what percent of the earth? What would be the measure of the intercepted arc?
2. Investigation: A circular dinner plate is broken and must be replaced. Only a part of the plate is left and the circumference of the original must be determined. Use your knowledge of chords and their perpendicular bisectors to obtain the original measure. Draw if necessary. (**Page 83**)

3.



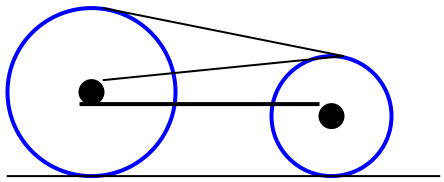
The diagram represents a church window. Find the area of the semi-circular arch.

4.



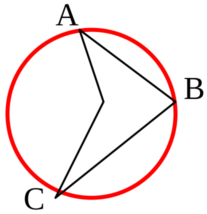
This is a company's logo. The square is 15 cm on each side. Find the measure of the chord.

5.



A toy designer started to create a new type of cycle. The wheel centers are 2 feet apart. The wheel circumferences are 1 foot apart. One wheel is 2 times bigger than the other. (a) What is the area of each wheel? (b) What distance on the ground will the cycle occupy?

6.



This figure represents a new car logo.

P is the center of the circle.
The radius of the circle is 14 m.
The central angle is 140º.

(a) Name the central angle.
(b) Name the inscribed angle and state its measure.
(c) Name 1 minor arc and state its linear and degree measure.

7. Create a design that includes a circle, tangents to the circle, secants, central and inscribed angles, chords, and sectors. Color it creatively. Make it your personal logo.

Solutions to pages 90, 92, and 93

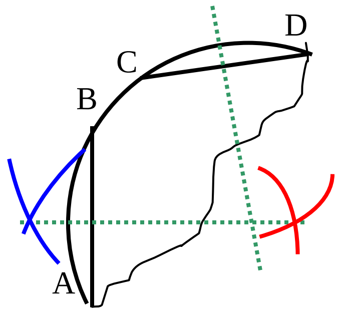
Page 90

1. x = 15 cm. Line segments from an external point tangent to a circle are congruent. Page 88.
2. No; the triangle should fit inside the circle and touch the circumference at each vertex.
3. Yes; C = 25 yards; A = 24 yards; $C^2 = a^2 + b^2$; $25^2 = 24^2 + x$; $625 - 576 = b^2$; $b = \sqrt{49}$; b = 7
4. Kite; The central angle = 115º; Explanation: RO and OT are congruent tangent segments from a common external point. Angle R ≡ Angle T = 90º. Subtract 65º + 90º + 90º from 360º to get 115º. The intercepted arc is congruent to the central angle.
5. If the radius of figure D is 21 inches, the area of the circumscribed polygon = S^2; S = 42m (same as the diameter). $42^2 = 1764\ m^2$.
6. Diagonal LN = measure of the hypotenuse of a 45-45 right triangle; using the pattern a, a, $a\sqrt{2}$ we get $42\sqrt{2}$ m.
7. Circumference = d π; if C = 20 π feet, then d = 20 feet. R = ½ d = 10 feet. $A = 10^2\ \pi = 100\ \pi$ sq feet.
8. Length of the hypotenuse of an isosceles right triangle = $a\sqrt{2}$; if a = 10 feet, $a\sqrt{2} = 10\sqrt{2}$ feet.

9. The hypotenuse of a 30-60-right triangle = 2a. If the diameter of the larger triangle = 20 feet we can solve for a by substitution.
A $\sqrt{3} = 20$; $a = 20/\sqrt{3}$ and $2a = \frac{2 \cdot 20 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{40\sqrt{3}}{3}$
10. QR is a **common external tangent**. The angles formed at the point of tangency **equal 90º**.

Page 92, real-world applications

1. Common external tangents; (b) Vertical angles; right angles to the radii at the point of tangency; (c) Diameter; (d) The point of tangency; 180º
2. The perpendicular bisector of a chord intersects the center of a circle. Imagine the broken plate to be an arc AB. Draw 2 chords AB and CD. Bisect both chords. They should intersect at the center of the circle. Find any radius from that construction and calculate the circumference.

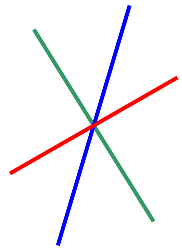


3. The diameter of the arch is 14 feet. The radius is 7 feet.
Area = 7 · 7 · 22/7 = 154 sq feet. Pi = 22/7 and is effective when the radius or diameter are multiples of 7.
4. The diagonal of the square is the chord referred to in the diagram. If the side of the square is 15 feet the diagonal is 15 $\sqrt{2}$.
5. If the wheel centers are 2 feet apart and their circumferences are 1 foot apart, then there is **one foot** of length for both radii. One wheel is 2 times bigger than the other; one wheel is x and the other wheel is 2x. Let's make an equation for this part. 2x + x = 1 foot or 12 inches. 3x = 12 inches; x = 4 inches. One wheel is 4 inches; the other is 8 inches. (a) Area of the smaller wheel = 16 π square inches; area of the larger wheel is 64 π square inches. (b) Add the distance between the wheel centers, and the radius of each wheel: 2feet + 4 inches + 8 inches = 3 feet.
6. (a) APC = central angle (b) ABC = inscribed angle (c) AC is a minor arc with a degree measure of 140º and a linear measure of 140/360 · 14 · 14 · 22/7 = 239.5 m^2. The inscribed angle is one half of the central angle and the intercepted arc.
7. On your own. Have Fun!

Points and Lines of Concurrency

Points of concurrency are specific points where 3 or more lines intersect.

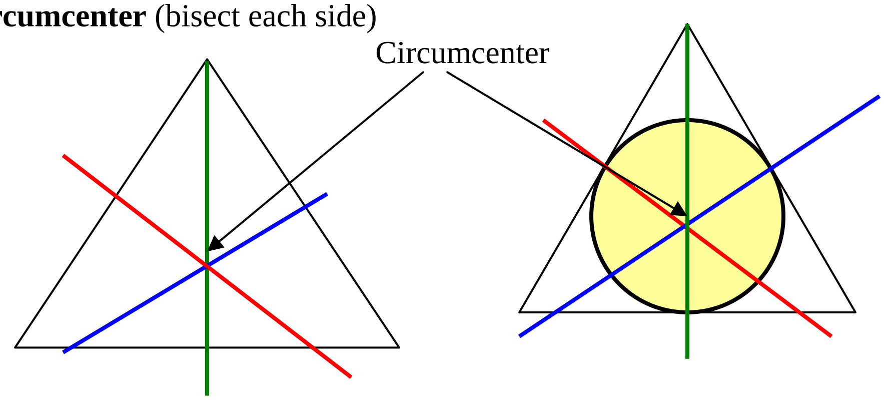
Concurrent means happening at the same time.



In geometry we work with 3 special lines of concurrency that meet a t 3 special points of concurrency.

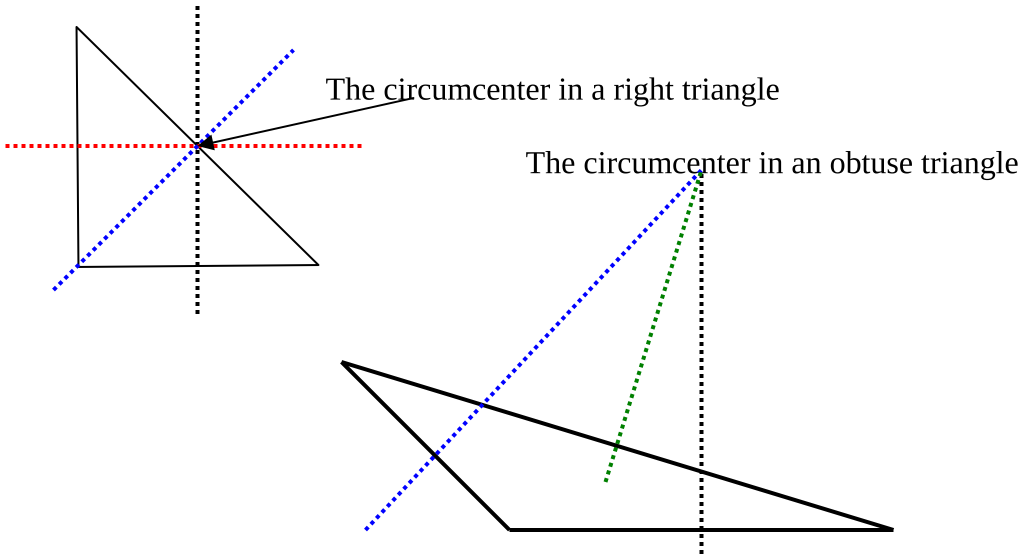
The **circumcenter** is the intersection of 3 perpendicular bisectors
The **incenter** is the intersection of 3 angle bisectors
The **centroid** is the meeting point of 3 medians in a triangle
The **orthocenter** is the place where 3 altitudes meet.

Constructing the circumcenter (bisect each side)

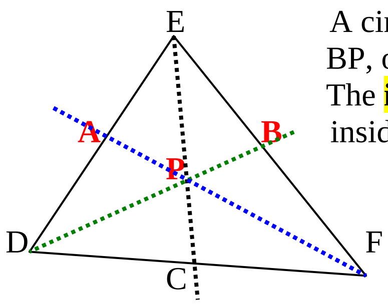


How does knowledge of this point help? Using this point helps us to inscribe a circle completely inside the triangle.

- Each line segment from the circumcenter to the mid-point of each side is a radius.
- The circumcenter will allow us to circumscribe a circle around the triangle because it defines the center of the triangle.
- The circumcenter will be inside of an equilateral or an isosceles triangle, on the hypotenuse of a right triangle, and outside of an obtuse triangle.

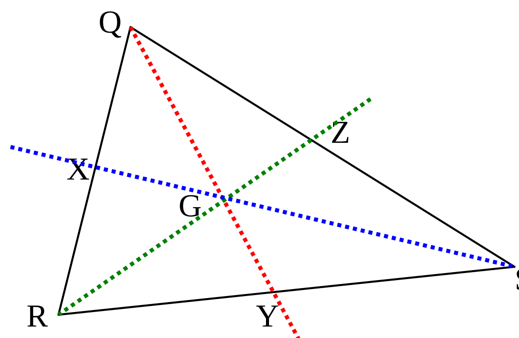


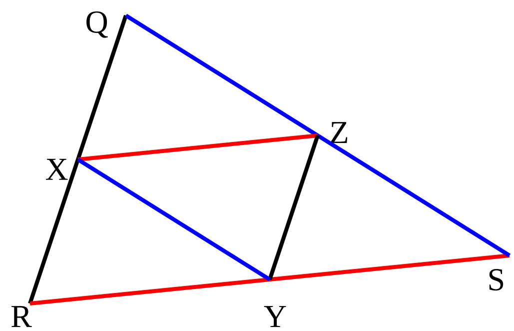
Constructing the incenter (bisect each angle)



P is the incenter of triangle DEF. AP ≡ BP ≡ CP ≡ radii
A circle can be inscribed inside this triangle with radii AP, BP, or CP.
The **incenter** and **circumcenter** can be used to inscribe circles inside a triangle.

Constructing the centroid
Find the mid-point of each base; connect the mid-point to each vertex



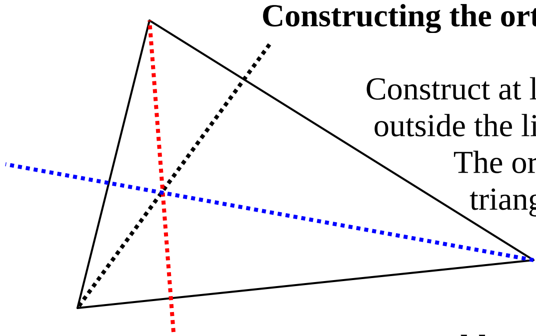


QRS is an acute triangle with medians QY, RZ, and SX. Y is the mid-point of RS, Z is the mid-point of QS, and X is the mid-point of QR. G is the point of concurrency called the centroid. **The centroid is usually named G because it defines the center of gravity**. Toys and objects that balance on a point are actually balancing on the centroid.

Interesting facts about the centroid:

- The distance from the centroid to the mid-point of each side is one-third of its entire length. GY = 1/3 of QY; GZ = 1/3 of RZ
- The distance from the centroid to each vertex is 2/3 the length of the centroid. GQ = 2/3 of QY; GR = 2/3 of RZ.
- Points X, Y, and Z form another triangle. Δ XYZ is ¾ of the area of Δ QRS. Observe that joining the mid-points creates 4 similar triangles. Each small triangle is ¼ of the larger triangle.
- Each median is parallel to a base. **XZ is parallel to RS**; **XY is parallel to QS**; **YZ is parallel to RQ.**
- The previous two bullets define the triangle mid-segment theory that states that the mid-segments of a triangle are parallel to the bases.

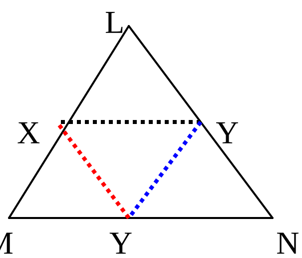
Constructing the orthocenter

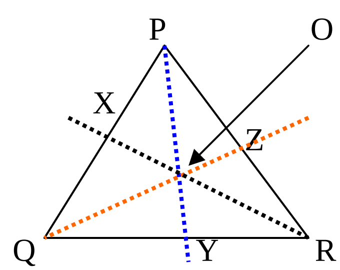


Construct at least 2 altitudes (drop a perpendicular from a point outside the line).
The orthocenter is inside the triangle is it is an acute triangle. If the triangle is a right triangle, the orthocenter is a vertex.

Problems! Problems! Problems!

1. Name any lines of concurrency that you could use to inscribe a circle inside a triangle.
2. Which line of concurrency comes from the construction of perpendicular bisectors?
3. Why are the circumcenter and the incenter so alike?
4. A circle inscribed in a triangle intersects the triangle at 3 points. How would you define each side of the triangle? What would you call the point where the circle intersects the triangle?
5. Angle bisectors create ____
6. The line segment connecting the circumcenter to the mid-point of each side of the triangle is a _____________________ of the circle.
7. In an obtuse the circumcenter would be __________________ of the circle.
8. What is the incenter?
9. Why is the centroid sometimes letter-named G?
10. What is the centroid?
11. The centroid divides the line segment it creates into 2 parts. What is the ratio of the longer to the shorter part?
12. What polygon is created when the mid-points are connected?
13. What is the ratio of the inner triangle to the original triangle?
14. Where is the orthocenter located in a right triangle?
15. What is an orthocenter?
16.





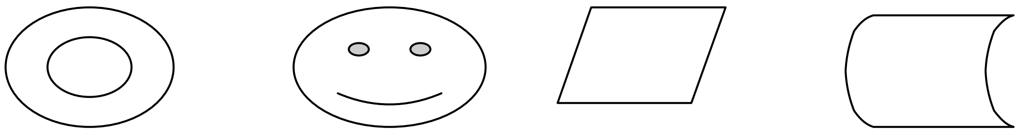
LMN is a triangle with the mid-points connected. PQR is a triangle showing the 3 medians and the centroid. O is the centroid.
(a) PY = 15 cm. What is the measure of PO and YO?
(b) RX = 18 cm. Find OX.
(c) ZO = 4 cm. Find ZQ.
(d) QR = 9 cm. What is the measure of QY?
(e) PQ = 9.5 cm. How long is QX?

Solutions to page 97

1. Circumcenter, incenter
2. Circumcenter
3. They can both inscribe circles from a center inside a triangle.
4. Each side of the triangle is a tangent. The circle intersects the triangle at a point of tangency.
5. Incenters
6. Radius.
7. Outside
8. The point at which 3 angle bisectors intersect.
9. It is the center of gravity.
10. The point at which 3 medians intersect.
11. The ratio of the longer part of the centroid to the shorter part is 2:1
12. Triangle.
13. The ratio of the inner triangle to the original triangle is 1:4
14. In a right triangle the orthocenter is located at a vertex.
15. The intersection of 3 altitudes.
16. (a) PO = 10 cm and YO = 5 cm
    (b) OX = 6 cm
    (c) ZO = 12 cm
    (d) QY = 4.5 cm
    (e) QX = 4.75 cm

Lesson 7-C
Review and Self-check
Units 6 and 7

1. What geometric shapes create the best tessellations?
2. List 2 letters of the alphabet whose upper cases have vertical symmetry, 2 that have horizontal symmetry, 1 that has diagonal symmetry, 2 that have both horizontal and vertical symmetry, and 1 that has horizontal, vertical, and diagonal symmetry.
3. Complete by inserting horizontal, vertical, or diagonal symmetry as appropriate:



________________ ________________ ________________ ________________

4. Use compass and straight edge to construct a 30º-60º right triangle. **Clue: Start with an equilateral.**
5. List 4 Pythagorean Triples.
6. Complete each missing leg of the right triangle: (a) 13, 84, ____________ (b) 7, ________, 25; (c) _________, 35, 37; (d) ________, 48, 50
7. Construct one of them using compass and straight edge.
8. State T or F: √36 + √64 = (√36 + 64)
9. Solve using radicals as much as possible: (a) √50 (b) √12 (c) √45
10. Complete: If the square of leg C is equal to the ____, then the triangle is a right triangle.
11. Is this a right triangle? (a) 18, 80, 82 (b) 11, 60, 60 (c) 9, 9, 12.72
12. State T or F: Square roots may be positive or negative.
13. Solve using radicals as much as possible: (a) $(\sqrt{7})^2$ (b) $(5\sqrt{7}) \cdot (3\sqrt{7})$ (c) $(4\sqrt{4}) + (2\sqrt{9})$ (d) $\sqrt{81} \cdot \sqrt{27}$
14. If leg a of an isosceles right triangle is 10 cm, what is leg b or leg C?
15. If leg C of a 30º-60º-90º triangle is 30 cm, what is the measure of leg a and leg b?
16. A triangle measured 5, 12, and 13 m on legs a, b, and C respectively. If AB measured 12 cm, what is the sine, cosine, and tangent of angle A?
17. Refer to the triangle at #23. What is the cosine and tangent of angle B?
18. Is the sine of 60º the same as 2 sin 30º?
19. Use trig tables to find the degree measure of each angle: (a) sin a = .6561 (b) cos a = .0523 (c) tan 1.000 (d) sin a = .9659
20. How is a secant different from a tangent?
21. What is the relationship between an inscribed angle and a central angle?
22. The ratio of an arc to a central angle is ________________________
23. If a central angle is 57º what is the measure of the intercepted arc?
24. If a central angle is 270º what fraction of the circumference is the intercepted arc?
25. Complete: Concentric circles have the same ______________________________
26. The side of a square circumscribed around a circle is the same measure as the _________________ of the circle.
27. Tangents drawn to a circle from an external point are congruent to ____
28. An external tangent common to two circles touches the circles at ___________ points of tangency.

Solutions and Partial Solutions

Questions 1 and 2: Page 99. Vertical symmetry: A, H, I, M, U, V, W, X, Y; horizontal symmetry: B, C, D, E, H, K, X; diagonal symmetry: O; HV and D: O
3. Hand V, H; D; H. 5) See lesson 2 of unit 3 to construct an equilateral triangle; bisect one of the sides, and draw a median. 6) See lesson 1 of unit 7
7. (a) 13, 84, **85** (b) 7, **24**, 25 (c) **12**, 35, 37 (d) **14**, 48, 50
8. On your own 9) F; Explanation: √36 + √64 = 6 + 8 = 14;
(√36 + 64) = √100 = 10 10) (a) √50 = √25 · √2 = 5 √2 (b) √12 = √3 · √4 = 2 √3
(c) √45 = √5 · √9 = 3 √5
11. Complete: If the square of leg C is equal to the **square of leg a plus the square of leg b, then the triangle is a right triangle.** 12) (a) Yes (b) No (c) Yes 19) Yes
13. (a) 7 (b) 15 √7 (c) 8 + 6 = 14 (d) 9 x 3 = 27
14) Leg b = 10 cm; leg C = 10 √2 15) Leg a = 15 cm; leg b = 15 √3
16. Sin A = 5/13; Cos A = 12/13; Tan A = 5/13 17) Cos B = 5/13; Tan B = 13/12
18. No; **sin 60º = .8660** and **2 sin 30 = 2 times .5 or 1.000**
19) (a) 41º (b) 87º (c) 45º (d) 15º 20) A secant originates and terminates ***outside*** of a circle, passing through it like a chord, intersecting it in 2 places; a chord originates and terminates on the ***circumference*** passing through the circle and intersecting the circle in 2 places. 21) An inscribed angle is ½ of a central angle
22. 1:1 23) 57º 24) ¾ 25) Concentric circles have the same radius
26. The side of a square circumscribed around a circle is the same measure as the **diameter**. 27) Tangents drawn to a circle from an external point are congruent to **each other at the point of tangency** 28) An external tangent common to two circles touches the circles at **2** points of tangency.

COURSE OBJECTIVES

The purpose of this course is to develop the geometric relationships and deductive strategies that can be used to solve a variety of real world and mathematical problems. The student will:

- Understand geometric concepts such as perpendicularity, parallelism, tangency, congruency, similarity, reflections, symmetry and transformations including flips, slides, turns, enlargements, rotations and fractals.
- Analyze and apply geometric relationships involving planar cross-sections (the intersection of a plane and a three-dimensional figure).
- Use properties and relationships of geometric shapes to construct formal and informal proofs.
- Add, subtract, multiply and divide real numbers including square roots and exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil and calculator.
- Use estimation strategies in complex situations to predict results and to check the reasonableness of results.
- Use concrete and graphic models to derive formulas for finding parameter, area, surface area, circumference and volume of two – and three-dimensional shapes, including rectangular solids, cylinders, cones and pyramids.
- Use concrete and graphic models to derive formulas for finding rate, distance, time, angle measures and arc lengths.
- Relate the concepts of measurement to similarity and proportionality in real-world situations.
- Select and use direct (measured) and indirect (not measured) methods of measurement as appropriate.
- Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area and volume and estimate the effects of measurement errors on calculations.

- Represent and apply geometric properties and relationships to solve real-world and mathematical problems including ratio, proportion and properties of right triangle trigonometry.
- Understand geometric concepts such as perpendicularity, parallelism, tangency, congruency, similarity, reflections, symmetry and transformations including flips, slides, turns, enlargements, rotations and fractals.
- Using a rectangular coordinate system (graph), apply and algebraically verify properties of two – and three-dimensional figures, including distance, midpoint, slope, parallelism and perpendicularity.

LEADING TODAY FOR A BETTER TOMORROW

CONTINENTAL ACADEMY

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