Mathematics Skills 1

By: Idania Dorta

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CONTINENTAL ACADEMY

INSTRUCTIONS

Welcome to your Continental Academy course "Mathematics Skills 1". It is made up of 8 individual lessons, as listed in the Table of Contents. Each lesson includes practice questions with answers. You will progress through this course one lesson at a time, at your own pace.

First, study the lesson thoroughly. Then, complete the lesson reviews at the end of the lesson and carefully check your answers. Sometimes, those answers will contain information that you will need on the graded lesson assignments. When you are ready, complete the 10-question, multiple choice lesson assignment. At the end of each lesson, you will find notes to help you prepare for the online assignments.

All lesson assignments *are* open-book. Continue working on the lessons at your own pace until you have finished all lesson assignments for this course.

When you have completed and passed all lesson assignments for this course, complete the End of Course Examination.

If you need help understanding any part of the lesson, practice questions, or this procedure:

- **Click on the "Send a Message" link on the left side of the home page**
- **Select "Academic Guidance" in the "To" field**
- **Type your question in the field provided**
- **Then, click on the "Send" button**
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About the Author…

Mrs. Idania Dorta earned her Bachelor of Science Degree in Mathematics Education from Florida International University and her Master of Science Degree in Computer Applications in Education from Barry University. She worked for the Dade County Public Schools System from 1993 until 2003, first as a classroom teacher, then as a mathematics department head and finally as the District Mathematics Educational Specialist. Since 2003, she has worked as an independent Mathematics Consultant. Mrs. Dorta is a veteran of numerous seminar presentations for educators and students alike. Idania makes her home in Miami, Florida with her husband and child.

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The purpose of this course is to provide experiences in problem solving, reasoning, and connections in mathematics. The content includes order of operations and operations with integers, decimals and fractions (positive and negative).

- Student will understand meanings of operations and how they relate to one another
- Students will compute fluently and make reasonable estimates

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Lesson 1 Place Value

Lesson 1a: **Whole Numbers and Decimals**

Each digit in a number represents a place value. The place value tells us how much that digit represents. In 45,762, the five represents 5 thousand because the five is in the thousands place.

Let's begin by taking a look at a place value chart for whole numbers. In the chart, each group of three digits is called a period. This is helpful when reading numbers. For example, 45,762 is read as: 45 thousand, 762. Notice that since 45 was in the thousands period the name of the period was read after 45. We don't read the ones period because it is understood. Read the following numbers in the place value chart aloud.

	Hundred Billions	Ten Billions	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
1.								4	5	7	6	2
2.			1	2	0	0	5	6	9	0	8	0
3.		4	0	3	7	7	8	1	0	0	0	0
4.					7	6	9	2	1	9	3	5
	Billions			**Millions**			**Thousands**			**Ones**		

Numbers are written in three different ways.

Standard form: 45,762

Word form: forty-five thousand, seven hundred sixty-two

Number-word form: 45 thousand, 762

Write examples **2**, **3**, and **4** on your own in Standard form, Word form, and Number-word form and check your answers below.

Example 2:

Standard form: 1,200,569,080

Word form: one billion, two hundred million, five hundred sixty-nine thousand, eighty

Number-word form: 1 billion, 200 million, 569 thousand, 80

Example 3:

Standard form: 40,377,810,000

Word form: forty billion, thr ee hundred seventy-seven million, eight hundred ten thousand

Number-word form: 40 billion, 377 million, 810 thousand

Example 4:

Standard form: 76,921,935

Word form: seventy-six million, nine hundred twenty-one thousand, nine hundred thirty-five

Number-word form: 76 million, 921 thousand, 935

Notice that, in each period, we write a hyphen (-) between the ten and ones place, i.e., seventy-six, and that, after each coma, we say the name of each of the periods (except for the ones period).

Now let's take a look at a place value chart for whole num bers **and** decimals. We use this to help read and write numbers smaller (as well as larger) than one.

	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths	Ten Thousandths
1.				**6**	**2**	**5**	**.**	**5**	**4**		
2.			**5**	**9**	**8**	**3**	**.**	**0**	**2**	**4**	
3.					**1**	**0**	**.**	**0**	**0**	**3**	**7**

The first thing to notice about the table is that the decimal point separates the whole numbers from the decimals, and that the place values after the decimal end in –**ths** to denote the difference between the **tens** place (to the left, larger than one) and the **tenths** place (to the right, smaller than one). Let's take a look at the numbers in the place value chart.

Example 1 is read as six hundred twenty-five **and** fifty-four hundredths. Notice that 625 is read as a whole number greater than one, **the decimal point is read as "and"**, and 54 is read as a number, but we say the place value it ends with.

We can also write decimals in Standard form, Word form, and Number-word form. See **Example 1**.

Standard form: 625.54

Word form: six hundred twenty-five **and** fifty-four hundred**ths**

Number-word form: 625 **and** 54 hundred**ths**

You try **Examples 2** and **3**. Write each in Standard form, Word form, and Number-word form.

Example 2:

Standard form: 5,983.024

Word form: five thousand, nine hundred eighty-three **and** twenty-four thousand**ths**

Number-word form: 5 thousand, 983 **and** 24 thousand**ths**

Example 3:

Standard form: 10.0037

Word form: ten **and** thirty-seven **ten thousandths**

Number-word form: 10 **and** 37 **ten thousandths**

Notice that, in **Example 2,** we still included the comma to separate the thousands period from the ones period, and the last digit (4) is in the thousandths place so that is the place value noted. In **Example 3,** we named the place value of the last digit (7), which is in the ten-thousandths place.

Practice

1. What is the place value of the underlined digit in 234,<u>0</u>95,471?
   a. hundreds b. hundred thousandths c. hundred thousand d. millions

2. Which of the following represents three hundred twenty-five and ninety-seven thousandths in Standard form?
   a. 325.97 b. 325,097 c. 325 and 97 thousandths d. 325.097

3. What is the value of the 5 in 375,230?
   a. five hundred b. five thousandths c. five thousand d. fifty thousand

4. The average body length of a bug is 15 thousandths of an inch. Which of the following expresses this number as a decimal?
   a. 0.15 b. 0.015 c. 0.0015 d. 0.00015
5. Write 3 billion, 7 hundred thousand, 42 in standard form?
   a. 3,000,700,042 b. 3,700,042 c. 3,700,042 d. 3,000,700.42

Answers to practice:
1. c. hundred thousand
2. d. 325.097
3. c. five thousand
4. b. 0.015
5. a. 3,000,700,042

Lesson 1b:

Comparing and Ordering Whole Numbers and Decimals

When comparing and ordering whole numbers and decimals, three possibilities exist: the first number is larger than the second, the first number is smaller than the second, or the two are equal.

This is denoted by the following symbols: **> represents greater than**
< represents less than
= represents equal to

For example 5 is greater than 2, so we write 5 > 2.

When ordering larger numbers, we line up the digits according to their place value.

For example, to compare 62,609 and 62,783 , we first line up the digits and find the first place value beginning at the left in which the digits are different.

62,609

62,783

If you notice, the six is the same for both numbers in the ten thousands place. If we move on to the next place value, the thousands, these are the same as well. Therefore, we continue to move to the right in search of a differing digit in the same place value and compare those.

The hundreds place has a 6 in the first number and a 7 in the second number. These are different, so we can compare them. We know that 7 is greater than (>) 6, which means that 62,783 is larger than 62,609, or 62,609 is less than 62,783. This can also be written as:

$$62,609 < 62,783$$

When comparing and ordering several numbers, compare two numbers at a time.

Example 1: Order the following numbers from least to greatest: 243,679, 23,594, and 23,945.

Line up the first two numbers given: 243,679
23,594

Of these two 243,679 is larger since there is no digit in the hundred thousands place to compare to the 2 in 243,679. Compare 23,594 to 23,945.

(We chose to compare the smaller of the two we compared first: since we need to order the numbers from least to greatest.)

23,594

23,945

Looking at these two numbers, 23,594 is the smaller since 5 is less than (<) 9. In fact, 23,594 is the smallest of the three numbers we are comparing. So, we can order them in the following way: 23,594 < 23,945 < 243,679

Try the following examples on your own. And check your answers below.

Example 2: Compare 81,054 and 81,937 using <, >, or =.

If we line up the numbers: 81,054

81,937

9 is larger than 0, so 81,937 is the larger of the two.

Therefore, **81,054 < 81,937**

Example 3: Write 5 million, 260 million, and 1 billion in order from greatest to least using <, >, or =.

It may be simpler to see these numbers written in standard form.

5,000,000

260,000,000

1,000,000,000

Looking at the place value of these, the largest of the three is 1 billion. By comparing the remaining two, we see that 260 million is greater than 5 million, but smaller than 1 billion.

Written in order from greatest to least, we have

1,000,000,000 > 260,000,000 > 5,000,000

Comparing and ordering decimals is done in exactly the same way that it is done for whole numbers. The three possibilities still exist: greater than (>), less than (<) or equal to (=).

When comparing decimals, you may need to add zeroes to the right of the decimal point and this will not affect the value of the number.

For example, 12.3 = 12.30 = 12.300 = 12.3000…..

The value of the number continues to be 12 and 3 tenths.

Let’s look at an example where we may need to add zeroes to the right of the decimal in order to properly compare two numbers.

Example 1: Compare 103.5 and 103.537 using <, >, or =**.**

We will begin by lining up the numbers according to their place value.

103.5

103.537

We can see that the numbers are lined up by place value, but we don't have a digit to compare the three to. In this case, we can add zeroes to 103.5, while not changing its value, making it easier to compare.

103.5**00**

103.537

Now we can compare and see that 103.537 is larger than 103.5 and inversely 103.5 is smaller than 103.537. Therefore,

103.5 < 103.537

Try the following examples on your own. Check your answers below.

Example 2: Compare 15.75 and 15.8 using <, >, or =.

If we line up the numbers: 15.75

15.8

8 is larger than 7, so 15.8 is the larger of the two.

Therefore,

15.75 < 15.8

Practice

Compare the following using <, >, or =**.**

1. 638,279 ____ 638, 327

   a. <   b. >   c. =

2. 0.8291 ____ 0.82907

   a. <   b. >   c. =

List the following in order from least to greatest.

3. 57,388; 52,725; 15,752; 570,019
   a. 57,388 < 52,725 < 15,752 < 570,019
   b. 15,752 < 57,388 < 52,725 < 570,019
   c. 57,388 > 52,725 > 15,752 > 570,019
   d. 15,752 < 52,725 < 57,388 < 570,019

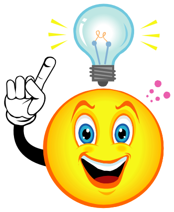
4. 11.066; 11.0666; 1.66
   a. 11.066 < 1.66 <11.0666
   b. 1.66 < 11.0666 < 11.066
   c. 1.66 < 11.066 < 11.0666
   d. 1.66 > 11.066 > 11.0666

5. The Smith's took a road trip and traveled 274.59 miles on Friday, 276.9 miles on Saturday, and 274.6 miles on Sunday. On what day did they travel the most?

a. Friday   b. Saturday   c. Sunday   d. none of these

Answers to practice:

1. **a.** <
2. **b.** >
3. **d.** 15,752 < 52,725 < 57,388 < 570,019
4. **c.** 1.66 < 11.066 < 11.0666
5. **b.** Saturday



LESSON 1 THINGS TO REMEMBER

Whole Numbers and Decimals

- Each digit in a number represents a place value.
- The place value tells us how much that digit represents. In 45,762, the five represents 5 thousand because the five is in the thousands place.

Let's begin by taking a look at a place value chart for whole numbers.

	Hundred Billions	Ten Billions	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
1.								**4**	**5**	**7**	**6**	**2**
2.			**1**	**2**	**0**	**0**	**5**	**6**	**9**	**0**	**8**	**0**
3.		**4**	**0**	**3**	**7**	**7**	**8**	**1**	**0**	**0**	**0**	**0**
4.					**7**	**6**	**9**	**2**	**1**	**9**	**3**	**5**
	Billions			**Millions**			**Thousands**			**Ones**		

In the chart, each group of three digits is called a period. This is helpful when reading numbers. For example, 45,762 is read as: 45 thousand, 762. Notice that since 45 was in the thousands period the name of the period was read after 45. We don't read the ones period because it is understood.

- Numbers are written in three different ways.

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Comparing and Ordering Whole Numbers and Decimals

- When comparing and ordering whole numbers and decimals, three possibilities exist:
    a. the first number is larger than the second,
    b. the first number is smaller than the second,
    c. or the two are equal.

This is denoted by the following symbols: **> represents greater than**

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For example 5 is greater than 2, so we write 5 > 2.

- When ordering larger numbers, we line up the digits according to their place value.

    For example, to compare 62,609 and 62,783, we first line up the digits and find the first place value beginning at the left in which the digits are different.

62,609
62,783

If you notice, the **six is the same** for both numbers in the ten thousands place. If we move on to the next place value, the thousands, these are the same as well. Therefore, we continue to move to the right in search of a differing digit in the same place value and compare those.

The hundreds place has a 6 in the first number and a 7 in the second number. These are different, so we can compare them. We know that 7 is greater than (>) 6, which means that 62,783 is larger than 62,609, or 62,609 is less than 62,783. **This can also be written as:**

62,609 < 62,783

- Comparing and ordering decimals is done in exactly the same way that it is done for whole numbers.
- The three possibilities still exist:

  greater than (>), less than (<) or equal to (=).
- When comparing decimals, you may need to add zeroes to the right of the decimal point and this will not affect the value of the number.

For example, 12.3 = 12.30 = 12.300 = 12.3000…..

<u>The value of the number continues to be 12 and 3 tenths</u>

Lesson 2: Decimals

In this lesson, you learn how to add, subtract, multiply, and divide decimal numbers.

Lesson 2a: Addition and Subtraction of Decimals

Adding and subtracting decimals is similar to adding and subtracting whole numbers except for one difference; before adding or subtracting, the decimals need to be lined up in the same way they lined up to compare and order them in Lesson 1b.

Let's look at an example of the addition of two decimals.

Example 1: Add 2.75 and 19.5678.

Line up the decimals first.

2.75**00**
+19.5678

Notice how zeroes are added to 2.75 as happens when comparing the place value with that of 19.5678.

Now just add.

2.75**00**
+19.5678

22.3178

Subtraction is done in the same way. It is especially important that you don't forget to add the zeroes in the missing places when subtracting due to borrowing if necessary.

Example 2: Subtract 15.431 from 22.9.

Line up the decimals making sure to list the larger one first.

22.9
- 15.431

Add zeroes to 22.9 to line up the place value with that of 15.431.
A **common mistake** is not to add zeroes and simply bring down the 1 and the 3. It is important to remember to add the zeroes to the 1 and 3 from ten in each place value.

Let's take a look at how this is done

89
22.900
-15.431
7.469

Try the next two examples on your own and check your answers below.

Example 3: Add 5.6, 1.97, and 4.631.

Line up the decimals (the order doesn't matter since we are adding).

5.6
1.97
+ 4.631

We add zeroes to line up the place values.

Finally, we add.

```
  5.600
  1.970
+ 4.631
 12.201
```

Example 4: Subtract 4.45 from 8.3.

Line up the decimals making sure to list the larger one first.

```
  8.3
- 4.45
```

We add zeroes to line up the place values. Make sure to borrow if necessary.

```
  7 2
  8.30
- 4.45
  3.85
```

Practice

Add or subtract the following.

1. 50.11 – 8.48

   a. 41.6 b. 41.63 c. 40 d. 42

2. $116.77 + $78.85

   a. $196 b. $200 c. $195.62 d. $195.60

3. 68.5 + 67.7 + 67.69 + 67.597

   a. 340 b. 269 c. 271.487 d. 272

4. 70.7 – 26.309 + 36

   a. 8.391 b. 133.009 c. 80.391 d. 106.7

5. Mario has 0.25 cup of mi lk, 0.33 3 cup of water, and 0.01 cup of chocolate. When he mixes the i ngredients, how much liquid does he have?

   a. 0.359 cup b. 1.143 cups c. 0.55 cups d. 0.593 cup

Answers to practice:

1. **b. 41.63**
2. **c. $195.62**
3. **c. 271.487**
4. **c. 80.391**
5. **d. 0.593 cup**

Lesson 2b: Multiplication and Division of Decimals

Multiplying decimals is similar to multiplying whole numbers. Remember that multiplication is repeated addition. Multiplying a whole number by a decimal is the same as repeatedly adding the decimal to itself the same number of times as the whole number. Keep in mind that the numbers we are multiplying are called factors and the answer to the multiplication problem is the product.

Let’s look at an example where we multiply a whole number by a decimal.

Example 1: Multiply 23.7 by 394.

We don't line up the decimals as we do when we are adding or subtracting.

$$\begin{array}{r} 394 \\ \underline{\times\ 23.7} \end{array}$$

Multiply each digit as you would multiply 394 by 237.

$$\begin{array}{r} 394 \\ \underline{\times\ 23.7} \\ 93378 \end{array}$$

Now the only remaining step is to count how many digits we have to the right of the decimal point in each of the two factors. Since 394 is a whole number we have no digits to the right of the decimal point except for 0, but 23.7 has 1 digit to the right of the decimal point. We place a decimal point at the end of 93378 and move one digit to the left, accounting for the 1 digit to the right of the decimal in 23.7.

$$\begin{array}{r} 394 \\ \underline{\times\ 23.7} \\ 9337\ 8 \end{array}$$

So our final product is 9337.8.

Let's look at an example where we multiply a decimal by another decimal. The same rules still apply, but we must account for the amount of digits to the right of the decimal point in each of the factors.

Example 2: Multiply 41.5 by 0.031.

We do not line up the decimals as we do when we are adding or subtracting.

$$\begin{array}{r} 41.5 \\ \underline{\times\ 0.031} \end{array}$$

Multiply each digit as you would multiply 415 by 31

$$\begin{array}{r} 415 \\ \underline{\times\ 31} \\ 12865 \end{array}$$

Count how many digits we have to the right of the decimal point in each of the two factors.

41.5 has one digit to the right of the decimal point and 0.031 has three digits to the right of the decimal point. In all we have 4 digits to the right of the decimal point.

We will place a decimal point at the end of 12865 and move four digits to the left accounting for the 4 digits to the right of the decimal in each of the factors.

$$\begin{array}{r} 41.5 \\ \underline{\times\ 0.031} \\ 1.2865. \end{array}$$

So our final product is 1.2865.

Let's try one more before you do some on your own.

Example 3: Multiply 0.2 by 0.375.

This would be the equivalent to multiplying

375
x 2
750

Now let's count the number of digits to the right of the decimal in each of the factors. 0.2 has one digit to the right of the decimal point and 0.375 has three. This means we will need to move our decimal point 4 spaces to the left of 750. Let's see.

0.375
x 0.2
750.

What we notice is that after moving the decimal point 4 spaces to the left we have run out of numbers, so we must add a zero to occupy that empty space. Remember, as we have learned in Lesson 1b, we can add zeroes to the right of the decimal point without affecting the value of the number.

0.375
x 0.2
.0750.

So our final product is 0.0750 or 0.075.

Try the next two examples on your own and check your answers below.

Example 4: Multiply 74.4 by 3.85

First we multiply

744

x 385

286,440

74.4 has one digit to the right of the decimal point and 3.85 has two. Since there is a total of 3 digits we place a decimal point at the end of 286,440 and move the decimal point to the left 3 spaces.

So our final product is 286.440 or 286.44

Example 5: Multiply 86.15 by 0.3.

Multiply

8615

x 3

25,845

86.15 has two digits to the right of the decimal and 0.3 has one. We will place a decimal point at the end of 25,845 and move it to the left three spaces.

So our final product is **25.845**.

Practice

Multiply the following.

1. 5.13 x 4

   a. 2.052 b. 205.2 c. 25.65 d. 20.52

2. 7.7 x 1.8

   a. 1.386 b. 14.86 c. 138.6 d. 13.86

3. 0.24 x 0.3

   a. 0.72 b. 0.072 c. 0.0072 d.0.00072

4. 9.79 x 4.19

   a. 410.201 b. 4.10201 c. 41.0201 d. 42.0201

5. The Smith's new car averaged 27.3 miles per gallon on a trip to the Grand Canyon. If the gas tank hol ds 16.5 gallons, how far can they go on ONE tank of gas?

   a. 4.5 miles b. 45.04 miles c. 450.45 miles d. 4504.5 miles

Answers to practice:

1. **d. 20.52**
2. **d. 13.86**
3. **b. 0.072**
4. **c. 41.0201**
5. **c. 450.45 miles**

Dividing decimals is similar to dividing whole numbers in that the number being divided is the dividend, the number you divide by is the divisor, and the answer is the quotient.

When you divide by a whole number, you break the dividend into groups of equal size.

The same is true when you divide decimals. There are two different scenarios when dividing decimals: **(1)** dividing a decimal by a whole number, and **(2)** dividing a decimal by a decimal.

Let's look at the first scenario.

Example 1: Divide 153.92 by 32.

The dividend is 153.92 (the number being divided) and 32 is the divisor (the number you divide by). Let's set this up, as we would divide a whole number by another whole number.

$32\overline{)153.92}$

In this case, let's take care of the decimal right away by bringing the decimal point up to the quotient and simply dividing as we would with whole numbers.

4.81

$32\overline{)153.92}$

-128

5

2 9

- 25 6

3 2

- 3 2

0

So our final quotient is **4.81**.

Let's see what dividing a decimal by a decimal looks like.

Example 2: Divide 0.427 by 6.1

The dividend is 0.427 and the divisor is 6.1.

$$6.1 \overline{)\,0.427}$$

The first step is to convert the divisor (6.1) into a whole number by moving the decimal to the right the amount of spaces necessary. Since 6.1 is six and one tenth, the decimal point needs to be moved only once to the right to make it 61 (in other words, multiply 6.1 by 10).

If we multiply the divisor by ten to make it a whole number, we must also multiply the dividend by 10 so that the value of the problem doesn't change.

*Note: the decimal place of the divisor determines what to multiply the dividend and divisor by (10, 100, 1000, etc.) or how many places (digits) to move the decimal point to the right.

So our new problem looks like this.

6.1. 0 $\overline{)\ .4.\,2\ 7}$

Now we can do as in **Example 1** where we brought the decimal point up to the quotient and simply divided.

```
            .07
6.1. ) 0.4. 2 7
       -  0
          4 2 7
        - 4 2 7
              0
```

So our final quotient is **.07** or **0.07.**

Try the next two examples on your own and check your answers below.

Example 3: Divide 87**.**4 by 0**.**38.

Let's begin by setting up the division problem.

```
0.38 ) 87.4
```

Next we need to take the divisor (0**.**38) and convert it to a whole number by movi ng the deci mal point **two** places to the right or multiplying by 100 (since the decimal reads 38 hundredth). Keep in mind that, if we move the decimal point tw o spaces to the right, then we must also multiply (move two places to the right) the dividend (87**.**4) by 100.

```
0.3 8. ) 87.4 0.
```

Notice how we had to add a zero after 4 to hold that additional place value.

Now we are ready to divide the dividend by the divisor.

```
            2 3 0 .
         ___________
0.3 8. ) 87.4 0.
        - 76
          114
        - 114
            0 0
          -   0
         _______
              0
```

So our final quotient is **230.** or **230**.

Example 4: Divide 0**.**7042 by 0**.**07.

Begin by setting up the problem and looking at the divisor to see how many spaces do we need to move the decima l point to the right. We see that we need to move it over 2 di gits (or mu ltiply by 100) because the deci mal reads 7 hundredths.

Keep in mind that, if we multipli ed the divi sor by 100, we had to do the same to the dividend. Begin dividing.

```
              1 0 . 0 6
           _______________
0. 0 7. ) 0. 7 0 . 4 2
           - 7
             0 0 4
             -   0
                 4 2
               - 4 2
                   0
```

So our final quotient is **10.06**.

Practice

1. Which of the following is the divisor in the division problem below?

2.618 ÷ 0.34 = 7.7

a. 2.618 b. 0.34 c. 7.7 d. none of these

Divide the following.

2. 103.82 by 50.

a. 2.0764 b. 20.764 c. 207.64 d. 2076.4

3. 1.5156 by 0.3

a. 0.05052 b. 0.5052 c. 5.052 d. 50.52

4. 0.76 by 0.4

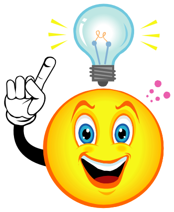
a. 2 b. 1.9 c. 19 d. 0.19

5. The Ferguson's traveled 7315 miles on 15.4 gallons of gasoline. How many miles per gallon did they get?

a. 4.75 b. 47.5 c. 475 d. 475.5

Answers to practice:

1. **b. 0.34**
2. **a. 2.0764**
3. **c. 5.052**
4. **b. 1.9**
5. **c. 475**



LESSON 2 THINGS TO REMEMBER

Addition and Subtraction of Decimals

- Adding and subtracting decimals is similar to adding and subtracting whole numbers except for one difference; before adding or subtracting, the decimals need to be lined up in the same way they lined up to compare and order them in Lesson 1b.

Let's look at an example of the addition of two decimals.

Example 1: Add 2.75 and 19.5678.

Line up the decimals first..

$$\begin{array}{r} 2.75\mathbf{00} \\ + \ \underline{19.5678} \end{array}$$

Notice how zeroes are added to 2.75 as happens when comparing the place value with that of 19.5678.

Now just add.

$$\begin{array}{r} 2.75\mathbf{00} \\ + \ \underline{19.5678} \\ 22.3178 \end{array}$$

- Subtraction is done in the same way. It is especially important that you don't forget to add the zeroes in the missing places when subtracting due to borrowing if necessary.

Example 2: Subtract 15.431 from 22.9.

Line up the decimals making sure to list the larger one first.

22.9
- 15.431

Add zeroes to 22.9 to line up the place value with that of 15.431.
A **common mistake** is not to add zeroes and simply bring down the 1 and the 3. It is important to remember to add the zeroes to the 1 and 3 from ten in each place value.

Let's take a look at how this is done

 89
 22.900
-15.431
 7.469

Multiplication and Division of Decimals

- **Multiplying decimals** is similar to multiplying whole numbers. Remember that multiplication is repeated addition. Multiplying a whole number by a decimal is the same as repeatedly adding the decimal to itself the same number of times as the whole number. Keep in mind that the numbers we are multiplying are called factors and the answer to the multiplication problem is the product.

Let's look at an example where we multiply a whole number by a decimal.

Example 1: Multiply 23.7 by 394.

We don't line up the decimals as we do when we are adding or subtracting.

```
   394
x  23.7
```

Multiply each digit as you would multiply 394 by 237.

```
   394
x  23.7
 93378
```

Now the only remaining step is to count how many digits we have to the right of the decimal point in each of the two factors.

Since 394 is a whole number we have no digits to the right of the decimal point except for 0, but 23.7 has 1 digit to the right of the decimal point. We place a decimal point at the end of 93378 and move one digit to the left, accounting for the 1 digit to the
right of the decimal in 23.7.

```
   394
x  23.7
 9337 8.
```

So our final product is 9337.8

Let's look at an example where we multiply a decimal by another decimal. The same rules still apply, but we must account for the amount of digits to the right of the decimal point in each of the factors.

Example 2: Multiply 41.5 by 0.031.

We do not line up the decimals as we do when we are adding or subtracting.

41.5
x 0.031

Multiply each digit as you would multiply 415 by 31

415
x 31
12865

Count how many digits we have to the right of the decimal point in each of the two factors.

41.5 has one digit to the right of the decimal point and 0.031has three digits to the right of the decimal point. In all we have 4 digits to the right of the decimal point.

We will place a decimal point at the end of 1285 and move four digits to the left accounting for the 4 digits to the right of the decimal in each of the factors.

41.5
X 0.031
1.2 8 6 5.

So our final product is 1.2865.

- **Dividing decimals** is simil ar to di viding whole numbers in that the number being divided is the dividend, the number you di vide by i s the divisor, and the answer is the quotient.

When you divide by a whole number, you break the dividend into groups of equal size. The same is true when you divide decimals.

- ❖ There are two different scenarios when dividing decimals:

  (1) dividing a decimal by a whole number, and **(2)** dividing a decimal by a decimal.

Let's look at the first scenario.

Example 1: Divide 153.92 by 32.

The dividend is 153.92 (the number being divided) and 32 is the divisor (the number you divide by). Let's set this up, as we would divide a whole number by another whole number.

$$32\overline{)153.92}$$

In this case, let's take care of the decimal right away by bringing the decimal point up to the quotient and simply dividing as we would with whole numbers.

```
         4. 81
   32)153. 9 2
     -128
         5      2   9
     - 25  6
           3 2
       -   3 2
             0
```

So our final quotient is **4.81**.

You can buy 12 oranges for $4.80. How much does one orange cost?

Take $4.80 and divide it by 12 oranges to get a result of $.40 per orange <u>or</u> 40 cents each.

Lesson 3: Number Theory

Greatest Common Factor and Least Common Multiple

In this lesson, you learn about factors, finding the factors of a number, finding the common factors of two or more numbers, and selecting the greatest of these common factors. You also learn about prime and composite numbers.

In addition, this lesson gives you the opportunity to learn about multiples and finding the least common multiple of two or more numbers.

These tools will, in turn, help you to be successful in the upcoming lesson on Fractions.

Let's begin by discussing factors. Factors are the numbers that, when multiplied together, give you a product. For example, in the multiplication problem 5 x 3 = 15, 5 and 3 are the factors and 15 is the product (or answer to the multiplication problem).

To find all the factors a number has, you must list all the different ways that you can obtain that number as a product.

> For example, to find all the factors of 8, find all the pairs of factors that, when multiplied together, give the product of 8.
>
> 1 x 8 = 8
>
> 2 x 4 = 8
>
> Therefore, the factors of 8 are 1, 2, 4.and 8.

Let's find all the factors of 18.

1 x 18 = 18

2 x 9 = 18

3 x 6 = 18

Therefore, the factors of 18 are 1, 2, 3, 6, 9, and 18.

Let's discuss common factors. Common factors are factors that are the same for two or more numbers.

For example, when looking for the common factors of 12 and 20, list the factors of 12, then list the factors of 20 and see which factors they have in common.

Factors of 12: 1 x 12 = 2

2 x 6 = 12

3 x 4 = 12

The factors of 12 are 1, 2, 3, 4, 6, and 12.

Factors of 20: 1 x 20 = 20

2 x 10 = 20

4 x 5 = 20

The factors of 20 are 1, 2, 4, 5, 10, and 20.

The factors that are common to both 12 **and** 20 are: 1, 2, and 4.
Therefore these are the common factors of 12 and 20.

The process for finding the **G**reatest **C**ommon **F**actor (**GCF**) of two or more numbers is the same. In the previous example, the Greatest Common

Factor or GCF was 4, since 4 was the greatest of the factors that both 12 and 20 had in common.

Using the same process used in the previous example, complete the next two examples on your own then check your answers below.

Example 1: Find the Greatest Common Factor of 30 and 45.

Let's first start out by listing the factors of 30 and then listing the factors of 45.

Factors of 30: 1, 2, 3, 5, 6, 10, 15, and 30.

Factors of 45: 1, 3, 5, 9, 15, and 45.

The common factors of 30 and 45 are: 1, 3, and 15, but the greatest of these is 15.

Therefore, 15 is the Greatest Common Factor of 30 and 45.

Example 2: Find the Greatest Common Factor of 12, 24, and 30.

Let's first start out by listing the factors of 12, then of 24 and finally listing the factors of 30.

Factors of 12: 1, 2, 3, 4, 6, and 12.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, and 24.

Factors of 30: 1, 2, 3, 5, 6, 10, 15, and 30.

The common factors of 12, 24, and 30 are: 1, 2, 3, and 6, but the greatest of these is 6. Therefore, 6 is the Greatest Common Factor of 12, 24 and 30.

You may also want to find out if a number is a factor of another. You can do this by listing the factors as you did in the previous examples, or by using divisibility rules. These divisibility rules are helpful to determine **if one** number is a factor of a larger number.

Divisibility Rules

A number is divisible by **2** if the digit in the ones place is an even number.
A number is divisible by **3** if the sum of the digits is divisible by 3.
A number is divisible by **4** if the number formed by the last two digits is divisible by 4.
A number is divisible by **5** if the digit in the ones place is either 0 or 5.
A number is divisible by **6** if the number is divisible by **2 and 3**.
A number is divisible by **8** if the number formed by the last three digits is divisible by 8.
A number is divisible by **9** if the sum of the digits is divisible by 9.
A number is divisible by **10** if the digit in the ones place is 0.

Note: When in doubt, you can divide the number by the factor, if the remainder is 0, then it is a factor. This is what it means for one number to be divisible by another, or is a factor of another.

Try some examples of divisibility rules on your own and check your answers with the book.

Example 3: Is 199 divisible by 9?

No, 199 is not divisible by 9 because 1 + 9 + 9 = 19 and 19 is not divisible by 9.

Example 4: Is 1,480 divisible by 5?

Yes, 1,480 is divisible by 5 because the last digit of 1,480 is 0.

Example 5: Is 534 divisible by 6?

We have two conditions to check here: (1) if the number is divisible by 2, and (2) if the number is divisible by 3.

534 is divisible by 2 since the digit in the ones place (4) is an even number.

534 is divisible by 3 since the sum of the digits 5 + 3 + 4 = 12 and 12 is divisible by 3.

Therefore, since both conditions checked, then **534 is divisible by 6**.

Another point to consider is whether a number is prime or composite.

A prime number is a whole number that has only two factors; 1 and itself. Some examples of prime numbers are: 3, 5, 7and11.

A composite number is a whole number that has more than two factors. Some examples of composite numbers are the examples earlier in the lesson, with several factors.

Note:1 is neither prime nor composite. When two composite numbers do not have any common factors, they are said to be relatively prime.

Now let's discuss multiples. A multiple of a number is the product of that number and another number.

For example, the multiples of 3 are: 3, 6, 9, 12, 15, 18, 21, …
Multiples are obtained by multiplying the number by 1, 2, 3, 4, 5, 6, 7, … and so on.
You can list as many multiples of a number as you need.

As in common factors, we can also find the common multiples of two or more numbers.

For example, to find the common multiples of 3 and 15, we would list the multiples of 3 and of 15. See which multiples are in both lists.. Let's take a look.
Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45 …
Multiples of 15: 15, 30, 45, 60, 75, 90, 105, 120, 135…
From what we have listed, 15 is a common multiple of 3 and 15. This is no surprise since 3 x 5 = 15, thus we know that 3 x 5 = 15 and 15 x 1 = 15.

We could list an extended amount of multiples or simply the first one that we find the numbers have in common. In doing so, we find the **L**east **C**ommon **M**ultiple (**LCM**), or the smallest multiple the numbers have in common.

Let's find the Least Common Multiple or LCM of 4 and 10.
List the multiples of 4: 4, 8, 12, 16, 20, 24, 26, 28, 32, 36, 40…
List the multiples of 10: 10, 20, 30, 40, 50, 60,70, 80, …

The LCM of 4 and 10 is 20. Although, if we continued to list multiples of 4 and 10, we would find more in common, 20 is the least of the common multiples.

Try the next two examples on your own and check your answers.

Example 6: Find the Least Common Multiple of 4 and 7.

List the multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, ...

List the multiples of 7: 7, 14, 21, 28, 35, 42, 49, 56, 63, …

The LCM of 4 and 7 is 28.

Note: In the previous example, 4 x 7 = 28 and 28 happens to be the LCM. This is not always the case. If you were looking for the LCM of 2 and 4, the LCM **would not** be 8 (2 x 4 = 8). Although 8 **is** a common multiple, it **is not** the least common multiple. The LCM of 2 and 4 is 4.

Example 7: Find the Least Common Multiple of 5, 8, and 20.

List the multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45, …

List the multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, …

List the multiples of 20: 20, 40, 60, 80, 100, 120, 140, …

The LCM of 5, 8, and 20 is 40.

Practice

1. Is 18 a prime or a composite number?

   a. prime   b. composite

2. Which one of the following numbers can be divided into 52?

   a. 2   b. 3   c. 6   d. 7

3. Which one of the following numbers can be divided into 135?

   a. 2   b. 3   c.4   d. 6

4. Which one of the following numbers can **NOT** be divided into 282?

   a. 2   b. 3   c. 4   d. 6

5. Which of the following factors is the Greatest Common Factor of 24 and 60?

   a. 2   b. 4   c. 6   d. 12

6. Which of the following factors is the Greatest Common Factor of 16, 32, and 50?

   a. 2   b. 4   c. 8   d. 16

7. By how many numbers is 45 divisible?

   a. 2   b. 4   c. 6   d. many

8. Which is the Least Common Multiple of 88 and 4?

   a. 1   b. 2   c. 4   d. 88

9. Which is the Least Common Multiple?

   a. 3 b. 6 c. 15 d. 3

10. Jonathan has Cross-Country practice every 5 days and Karate every 6 days. How often does he have Cross-Country and Karate on the same day?

   a. Every 5 days b. Every 6 days c. Every 15 days d.Every 30 days

Answers to practice:

1. b. Composite

2. a. 2

3. b. 3

4. c. 4

5. d. 12

6. a. 2

7. c. 6

8. d. 88

9. d. 30

10. d. Every 30 days

LESSON 3 THINGS TO REMEMBER

Greatest Common Factor and Least Common Multiple

- Factors are the numbers that, when multiplied together, give you a product. For example, in the multiplication problem 5 x 3 = 15, 5 and 3 are the factors and 15 is the product (or answer to the multiplication problem).

To find all the factors a number has, you must list all the different ways that you can obtain that number as a product.

For example, to find all the factors of 8, find all the pairs of factors that, when multiplied together, give the product of 8.

1 x 8 = 8

2 x 4 = 8

Therefore, the factors of 8 are 1, 2, 4.and 8.

Let’s find all the factors of 18.

1 x 18 = 18

2 x 9 = 18

3 x 6 = 18

Therefore, the factors of 18 are 1, 2, 3, 6, 9, and 18.

- Common factors are factors that are the same for two or more numbers.

For example, when looking for the common factors of 12 and 20, list the factors of 12, then list the factors of 20 and see which factors they have in common.

Factors of 12: 1 x 12 = 12

2 x 6 = 12

3 x 4 = 12

The factors of 12 are 1, 2, 3, 4, 6, and 12.

Factors of 20: 1 x 20 = 20

2 x 10 = 20

4 x 5 = 20

The factors of 20 are 1, 2, 4, 5, 10, and 20.

The factors that are common to both 12 **and** 20 are: 1, 2, and 4.
Therefore these are the common factors of 12 and 20.

- The process for finding the **G**reatest **C**ommon **F**actor (**GCF**) of two or more numbers is the same. In the previous example, the Greatest Common Factor or GCF was 4, since 4 was the greatest of the factors that both 12 and 20 had in common.

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Factors of 30: 1, 2, 3, 5, 6, 10, 15, and 30.

Factors of 45: 1, 3, 5, 9, 15, and 45.

The common factors of 30 and 45 are:1, 3, and 15, but the greatest of these is 15.

Therefore, 15 is the Greatest Common Factor of 30 and 45.

Example 2: Find the Greatest Common Factor of 12, 24, and 30.

Let's first start out by listing the factors of 12, then of 24 and finally listing the factors of 30.

Factors of 12: 1, 2, 3, 4, 6, and 12.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, and 24.

Factors of 30: 1, 2, 3, 5, 6, 10, 15, and 30.

The common factors of 12, 24, and 30 are: 1, 2, 3, and 6, but the greatest of these is 6. Therefore, 6 is the Greatest Common Factor of 12, 24 and 30.

- You may also want to find out if a number is a factor of another. You can do this by listing the factors as you did in the previous examples, or by using divisibility rules. These divisibility rules are helpful to determine **if <u>one</u>** number is a factor of a larger number.

<u>Divisibility Rules</u>

- A number is divisible by **2** if the digit in the ones place is an even number.
- A number is divisible by **3** if the sum of the digits is divisible by 3.
- A number is divisible by **4** if the number formed by the last two digits is divisible by 4.
- A number is divisible by **5** if the digit in the ones place is either 0 or 5.
- A number is divisible by **6** if the number is divisible by **2 and 3**.

- A number is divisible by **8** if the number formed by the last three digits is divisible by 8.
- A number is divisible by **9** if the sum of the digits is divisible by 9.
- A number is divisible by **10** if the digit in the ones place is 0.

Note: When in doubt, you can divide the number by the factor, if the remainder is 0, then it is a factor. This is what it means for one number to be divisible by another, or is a factor of another.

Try some examples of divisibility rules on your own and check your answers with the book.

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No, 199 is not divisible by 9 because 1 + 9 + 9 = 19 and 19 is not divisible by 9.

Example 4: Is 1,480 divisible by 5?

Yes, 1,480 is divisible by 5 because the last digit of 1,480 is 0.

Example 5: Is 534 divisible by 6?

We have two conditions to check here: (1) if the number is divisible by 2, and (2) if the number is divisible by 3.

534 is divisible by 2 since the digit in the ones place (4) is an even number.

534 is divisible by 3 since the sum of the digits 5 + 3 + 4 = 12 and 12 is divisible by 3.

Therefore, since both conditions checked, then **534 is divisible by 6**.

- Another point to consider is whether a number is prime or composite.

- ❖ A prime number is a whole number that has only two factors; 1 and itself. Some examples of prime numbers are: 3, 5, 7and11.
- ❖ A composite number is a whole number that has more than two factors. Some examples of composite numbers are the examples earlier in the lesson, with several factors.
- ❖ **Note: 1 is neither prime nor composite.** When two composite numbers do not have any common factors, they are said to be relatively prime.

Now let's discuss multiples. A multiple of a number is the product of that number and another number.

For example, the multiples of 3 are: 3, 6, 9, 12, 15, 18, 21, …
Multiples are obtained by multiplying the number by 1, 2, 3, 4, 5, 6, 7, … and so on.
You can list as many multiples of a number as you need.

- ❖ As in common factors, we can also find the common multiples of two or more numbers. Let's take a look.

For example, to find the common multiples of 3 and 15, we would list the multiples of 3 and of 15. See which multiples are in both lists..
Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45 …
Multiples of 15: 15, 30, 45, 60, 75, 90, 105, 120, 135…
From what we have listed, 15 is a common multiple of 3 and 15. This is no surprise since 3 x 5 = 15, thus we know that 3 x 5 = 15 and 15 x 1 = 15.

- ❖ We could list an extended amount of multiples or simply the first one that we find the numbers have in common. In doing so, we find the **L**east **C**ommon **M**ultiple (**LCM**), or the smallest multiple the numbers have in common.

Let's find the Least Common Multiple or LCM of 4 and 10.

List the multiples of 4: 4, 8, 12, 16, 20, 24, 26, 28, 32, 36, 40…

List the multiples of 10: 10, 20, 30, 40, 50, 60,70, 80, …

The LCM of 4 and 10 is 20. Although, if we continued to list multiples of 4 and 10, we would find more in common, 20 is the least of the common multiples.

Try the next two examples on your own and check your answers.

Example 6: Find the Least Common Multiple of 4 and 7.

List the multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, ...

List the multiples of 7: 7, 14, 21, 28, 35, 42, 49, 56, 63, …

The LCM of 4 and 7 is 28.

Note: In the previous example, 4 x 7 = 28 and 28 happens to be the LCM. **This is not always the case.** If you were looking for the LCM of 2 and 4, the LCM **would not** be 8 (2 x 4 = 8). Although 8 **is** a common multiple, it **is not** the least common multiple. The LCM of 2 and 4 is 4.

Example 7: Find the Least Common Multiple of 5, 8, and 20.

List the multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45, …

List the multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, …

List the multiples of 20: 20, 40, 60, 80, 100, 120, 140, …

The LCM of 5, 8, and 20 is 40.

Lesson 4: Fractions

In this lesson, you learn about fractions. You learn how to compare, order, add, subtract, multiply, and divide fractions. You need to use the techniques you learned in Lesson 1b to compare and order numbers. Also you need to be able to find the Greatest Common Factor and Least Common Multiple of two or more numbers as you did in Lesson 3.

Lesson 4a: Comparing and Ordering Fractions

When comparing and ordering fractions, you will use the same symbols (learned in Lesson 1) of greater than (>), less than (<) and equal to (=). You need to compare and order fractions with the same denominator, or with different denominators.

Remember: A fraction is made up of a numerator and a denominator as shown below.

$$\frac{\textbf{Numerator}}{\textbf{Denominator}}$$

Comparing Fractions when the denominators are the same

This is quite simple since all you have to do is look at the numerators of the fractions and compare those.

For example, when comparing $\frac{1}{4}$ and $\frac{3}{4}$, since the denominators are both 4, we are going to compare the numerators 1 and 3. Since 1 is less than 3, $\frac{1}{4}$ is less than $\frac{3}{4}$. Therefore, $\frac{1}{4} < \frac{3}{4}$.

Comparing Fractions when the denominators are different

There are several ways this can be done.

Method 1: Write equivalent fractions with a common denominator. Let's see how this is done. You need to use the concept of finding the Least Common Multiple of the denominators or the Least Common Denominator.

For example, when comparing $^5/_6$ and $^7/_8$, since the fractions have different denominators, we are going to write fractions equivalent to $^5/_6$ and to $^7/_8$.
We do this by finding the Least Common Multiple of the denominators or **LCD** (Least Common Denominator). The LCD for 6 and 8 is **24**.
(We listed the multiples for 6: 6, 12, 18, **24**, 30, 36, 42, 48…
and the multiples for 8: 8, 16, **24**, 32, 40, 48…).
Now we are going to create equivalent fractions.

$\frac{5}{6} = \frac{20}{24}$ Since 6 x 4 = 24, we need to multiply 5 x 4 = 20, to obtain an equivalent fraction.

Now we will do the same with $^7/_8$.

$\frac{7}{8} = \frac{21}{24}$ Since 8 x 3 = 24, we need to multiply 7 x 3 = 21, to obtain an equivalent fraction.

So now we can compare the new fractions $\mathbf{\frac{20}{24}}$ and $\mathbf{\frac{21}{24}}$.

We compare these two fractions as we did in the previous example.
Since, **20**< **21**, then **20/24**< **21/24** and **5/6** < **7/8**.
(Notice that, when we created the equivalent fractions,
5/6 became 20/24 and 7/8 became 21/24.)

Method 2: A second way to compare fractions with unlike denominators is to convert the fractions into decimals and compare them as we did in Lesson 1. Let's take a look at the same example using this method.

Let's begin by converting 5/6 and 7/8 into decimals.

5/6 = 0.8333 and **7/8 = 0.875**, since **0.875 is larger**, then **5/6** < **7/8**.

Method 3: Another way this can be done is by cross-multiplying the products. Let's see.

We are going to cross-multiply the denominator of one fraction with the numerator of the other and vice versa.

$$\frac{5}{6} \times \frac{7}{8}$$

8 x 5 = 40 and 6 x 7 = 42, since 40 < 42, then 5/6 < 7/8.

You may choose to compare and order fractions in any of the three methods shown above. It is especially important that you recall how to compare fractions using Method 1 since you will be using it when adding and subtracting fractions with unlike denominators in the next lesson.

When comparing mixed numbers that have different whole numbers, simply compare the whole numbers. If the whole numbers are the same, use the following example..

For example, compare $3\ {}^{3}/_{4}$ and $3\ {}^{5}/_{16}$.

Using Method 1, we compare ${}^{3}/_{4}$ and ${}^{5}/_{16}$ by making equivalent fractions, since the denominators are different. (Notice we don't discuss the whole number 3 since it is the same for both mixed numbers.)

We do this by finding the Least Common Multiple of the denominators or LCD (Least Common Denominator). The LCD for 4 and 16 is **16**.

(We listed the multiples for 4: 4, 8, 12, **16**, 20, 24, 28, 32, 36, 40, 44, 48

and the multiples for 16: **16**, 32, 48, 64, 80 …).

Now we are going to create fractions that are equivalent to $^3/_4$. and to $^5/_{16}$

$\frac{3}{4} = \frac{12}{16}$ Since 4 x 4 = 16, we need to multiply 3 x 4 = 12, to obtain a fraction that is equivalent to ¾. .

Now we compare this to the $^5/_{16}$.

Since, 12/16 > 5/16, then **3/4 > 5/16**. Therefore, $3^3/_4 > 3^5/_{16}$.

Using Method 2, we convert $\mathbf{3^3/_4}$ and $\mathbf{3^5/_{16}}$ into decimals and then compare.

$\mathbf{3^3/_4 = 3.75}$ and $\mathbf{3^5/_{16} = 3.3125}$. Since **3.75** > **3.3125**, $\mathbf{3^3/_4 > 3^5/_{16}}$.

Using Method 3, we can cross-multiply the denominator of one fraction with the numerator of the other and vice versa.

$$\frac{3}{4} \times \frac{5}{16}$$

16 x 3 = 48 and 4 x 5 = 20, since **48** > **20**, then **3/4** > **5/16**.

Therefore, $\mathbf{3^3/_4 > 3^5/_{16}}$.

Try some examples of comparing fractions and mixed numbers on your own and check your answers below..

Example 1: Compare **8/11** and **5/7**.

Using Method 2, we convert 8/11 and 5/7 into decimals and then compare. **8/11 = 0.7272**… and

5/7 = 0.71429.

Since **0.7272 is greater than 0.71429**, **8/11** > **5/7**.

Example 2: Compare **$6\,{}^{2}/_{7}$ and $6\,{}^{1}/_{9}$**.

Using Method 3, we can cross-multiply the denominator of one fraction with the numerator of the other and vice versa.

$$\frac{2}{7} \times \frac{1}{9}$$

9 x 2 = 18 and 7 x 1 = 7, since 18 > 7, then **2/7 >1/9**.

Therefore, **$6\,{}^{2}/_{7}$** > **$6\,{}^{1}/_{9}$**.

When ordering fractions and mixed numbers, we do so the same way we did in Lesson 1. We compare two fractions or mixed numbers at a time, and then we compare the third, fourth, and so on. Try the next example on your own and check your answer below.

Example 3: Order **½, ${}^{3}/_{8}$, ${}^{5}/_{8}$, and ¼** from least to greatest.

By looking at the fractions, we know that ${}^{3}/_{8} < {}^{5}/_{8}$ (because 3 < 5 and the denominators are the same).

Now all we have to do is find out how ½ and ¼ compare to these. Using Method 2, ½ > ${}^{3}/_{8}$ but **less than** ${}^{5}/_{8}$ because ½ = 0.5, **3/8 = 0.375, and 5/8 = 0.625**. If we were to order these from least to greatest ½ would be between ${}^{3}/_{8}$ and ${}^{5}/_{8}$ or ${}^{3}/_{8}$< ½ < ${}^{5}/_{8}$. Now we still need to compare ¼ to the rest of the fractions. ¼ **= 0.25** so it is less than all of the fractions.

Therefore, **¼ < ${}^{3}/_{8}$ < ½ < ${}^{5}/_{8}$**.

Practice

Compare each the following using >, <, or =.

1. 18/44 and 9/34

a. > b. < c. =

2. 12/20 and 16/30

a. > b. < c. =

3. **7 $^{1}/_{5}$** and **6 $^{4}/_{5}$**

a. > b. < c. =

4. Which of the following shows 2/3, 9/10, 7/8, and 3/4 in order from least to greatest?

a. 2/3 < 9/10 < 7/8 < 3/4
b. 2/3 < ¾ < 7/8 < 9/10
c. ¾ < 2/3 < 7/8 < 9/10
d. 9/10 < 7/8 < ¾ < 2/3

5. Four pieces of string have lengths 2 ¾, 7/8, 1 ¼, and 7/2 inches. List their lengths in order from longest to shortest?

a. 2 ¾ > 7/8 > 1 ¼ > 7/2
b. 7/8 > 1 ¼ > 2 ¾ > 7/2
c. 7/2 > 2 ¾ > 1 ¼ > 7/8
d. 7/2 > 1 ¼ > 7/8 > 2 ¾

Answers to practice:

1. **a.** >
2. **a.** >
3. **a.** >
4. **b.** 2/3 < ¾ < 7/8 < 9/10
5. **c.** 7/2 > 2 ¾ > 1 ¼ > 7/8

Lesson 4b: Adding and Subtracting Fractions

When **adding or subtracting fractions with the <u>same</u> denominators**, you simply add or subtract the numerators and keep the denominators the same.

For example, 1/5 + 2/5.

Add the numerators 1 + 2 = 3, so 1/5 + 2/5 = 3/5.

Note that, in some cases, it may be necessary to simplify the answer.

When **adding or subtracting fractions with <u>different</u> denominators**, you must first change each fraction to an equivalent fraction with the same denominator (as seen in the previous lesson, Method 1) and then you can find the sum (+) or difference (-) as seen in the previous example.

For example, 2/3 + 7/8.

Using Method 1, we find the Least Common Multiple of the Denominators or LCD. The LCD for 3 and 8 is **24**.

(We listed the multiples for 3: 3, 6, 9, 12, 15, 18, 21, **24**, 27, 30 …
and the multiples for 8: 8, 16, **24**, 32 …).

Now we are going to create equivalent fractions.

$\frac{2}{3} = \frac{16}{24}$ Since 3 x 8 = 24, we need to multiply the 2 by 8 also, to obtain a fraction that is equivalent to 2/3.

Now we will do the same with 7/8.

$\frac{7}{8} = \frac{21}{24}$ Since 8 x 3 = 24, we need to multiply the 7 by 3 also, to obtain a fraction that is equivalent to 7/8.

Now we can add the equivalent fractions with the same denominator.

16/24 + 21/24 = 37/24

37/24 is an improper fraction and needs to written as a mixed number, so we divide 37 by 24 and get a result of 1 with a remainder of 13. Therefore, 2/3 + 7/8 = 1 $^{13}/_{24}$.

(Note that the whole number one is the result, the numerator 13 is the remainder, and 24 is the same denominator.)

Now we must simplify: 1 $^{13}/_{24}$, we simplify the fraction by finding the Greatest Common Factor of 13 and 24. This factor is 1 (13 is a prime number; factors are 1 and 13). This means that we can divide both the numerator and denominator by 1, leaving the fraction to be $^{13}/_{24}$

Therefore 2/3 + 7/8 = 1 $^{13}/_{24}$.

Adding and subtracting mixed numbers with the same and different denominators is the same as adding and subtracting fractions with the same and different denominators. Keep in mind that you must add or subtract the whole numbers in the mixed numbers as well as the fractions themselves, and sometimes you will need to simplify an improper fraction in you answer as in the previous example.

Try the following examples on your own and check your answers below.

Example 1: Subtract 5/8 – 1/6.

Before subtracting the numerators, we must create equivalent fractions so that the denominators are the same. Refer to Method 1.to create equivalent fractions.

5/8 = 15/24

Since 8 x 3 = 24, we multiply the 5 by the 3 also, to obtain a fraction equivalent to 5/8.

Doing the same for 1/6: 1/6 = 4/24 Since 6 x 4 = 24, we multiply the 1 by the 4 also, to obtain a fraction equivalent to 1/6

Now we can subtract 15/24 – 4/24. 15 – 4 = 11,

so our answer is 11/24.

(Since 11 and 24 do not have a common factor other than 1, 11/24 is in its lowest terms, cannot be simplified, and therefore is the final answer.)

Example 2: Add **23 $^{7}/_{10}$ + 37 $^{3}/_{10}$.**

To add mixed numbers with the same denominator, you just need to place the sum of the numerators over the common denominator. Then add the whole numbers.

7/10 + 3/10 = 10/10 = 1

23 + 37 = 60 + 1 = 61

(Since the sum of the fractions was 10/10 = 1, we do not have a fraction in our answer. Notice how we simply added the sum of the fractions to the sum of the whole numbers to obtain our final answer).

Example 3: Add **11 $^{2}/_{3}$ + 10 $^{4}/_{5}$**.

Let's begin by creating equivalent fractions for 2/3 and 4/5 since the denominators are different.

2/3 = **10/15**

(we chose 15 as the LCD because it is the LCM of 3 and 5).

Since 3 x 5 = 15, then multiply 2 by the 5 = 10.

4/5 = **12/15**. Since 5 x 3 = 15, then multiply the 4 by 3 = 12.

Now we can add the new fractions

10/15 + **12/15** = **22/15**.

This is an improper fraction; therefore, it needs to be simplified. We divide 22 by 15 and the result is 1 with a remainder of 7. Therefore, 22/15 = **1 7/15**. (Remember how this was calculated earlier: the 15 is the same denominator of the equivalent fractions. And 7 and 15 do not have any factors in common, except 1, so 7/15 cannot be simplified)

Now we add the sum of the mixed numbers 11 + 10 = **21**

to the sum of the fractions **1 $^{7}/_{15}$** + **21** = 22 $^{7}/_{15}$.

And **22 $^{7}/_{15}$** is the final answer.

The last topic to discuss in this section is **subtraction of mixed numbers when you need to borrow or have to rename the mixed number**.

Let's look at an example.

Subtract **$9\,^{1}/_{5} - 4\,^{4}/_{5}$.**

In this example you don't have to find the common denominator because they are the same, but you cannot subtract 4 from 1 so you have to borrow from the 9 or rename. Recall that **$9 = 8\,^{5}/_{5}$** because $^{5}/_{5}$ = 1 + 8 = 9. The reason you choose $^{5}/_{5}$ instead of any other fraction is because the common denominator of the two fractions is 5 and $^{5}/_{5}$ = 1. If the common denominator had been 23, you would choose **23/23** because **23/23** = 1.

Now add the 8 **$^{5}/_{5}$** to the **$^{1}/_{5}$** of the 9 **$^{1}/_{5}$** to rename the fraction. **$^{5}/_{5} + ^{1}/_{5} = ^{6}/_{5}$**

So **9 1/5** becomes **$8\,^{6}/_{5}$**.

Now you can subtract **$8\,^{6}/_{5} - 4\,^{4}/_{5}$.**

Subtract the fractions first **$^{6}/_{5} - ^{4}/_{5} = ^{2}/_{5}$**

and then the whole numbers **8 – 4 = 4**.

So your final answer is **$4\,^{2}/_{5}$.**

Try the next example on your own and check your answer below.

Example 4: Subtract **12 – 8 ¾.**

In this case you have a mixed number subtracted from a whole number. Notice that it would be WRONG to subtract 12 – 8 = 4 ¾. WHY?? BECAUSE, AFTER you take 8 (pies) away from 12 (pies), leaving 4 (pies), you STILL need to take away an additional ¾ (of a pie). Four whole pies will NOT be left in the kitchen, and CERTAINLY NOT 4 pies PLUS ¾ of another pie.

Rename 12 as 11 $^{4}/_{4}$ because $^{4}/_{4}$ = 1 + 11 = 12.

Now subtract **$11\,^{4}/_{4}$ – 8 ¾.** Begin by subtracting the fractions **$^{4}/_{4}$ – ¾ = ¼.** Now subtract the whole numbers **11 – 8 = 3**,

so our final answer is **3 ¼.**

Practice

Add or subtract the following fractions and mixed numbers.

1. 2/8 + 3/8

   a. $^{5}/_{8}$ b. $^{1}/_{8}$ c. 1 d. $^{5}/_{16}$

2. 2/3 – 5/8

   a. $^{30}/_{24}$ b. $^{1}/_{24}$ c. 1 $^{6}/_{24}$ d. 1 ¼

3. 22 ¾ + 19 $^{2}/_{5}$

   a. 41 $^{5}/_{9}$ b. 41 $^{23}/_{20}$ c. 42 $^{3}/_{20}$ d. 41 $^{3}/_{20}$

4. 7 $^{3}/_{8}$ – 2 $^{2}/_{3}$

   a. 4 $^{17}/_{24}$ b. 4 $^{7}/_{24}$ c. 5 $^{1}/_{5}$ d. 5 $^{23}/_{24}$

5. Marie has a box of macaroni and cheese that contains 6 servings. She plans to eat 3 $^{1}/_{3}$ servings. How many servings will be left?

   a. 3 $^{1}/_{3}$ b. 3 $^{2}/_{3}$ c. 2 $^{2}/_{3}$ d. 3 $^{2}/_{3}$

Answers to practice:

1. a. $^{5}/_{8}$ **2. b.** $^{1}/_{24}$ **3. c.** 42 $^{3}/_{20}$ **4. a.** 4 $^{17}/_{24}$ **5. c.** 2 $^{2}/_{3}$

Lesson 4c: Multiplying and Dividing Fractions

When **multiplying fractions** and dividing fractions, you do not need to find the common denominator as you did in addition and subtraction. All you need to do is multiply numerator by numerator and denominator by denominator, then simplify the fraction if necessary.

$$\frac{\text{Numerator}}{\text{Denominator}} \times \frac{\text{Numerator}}{\text{Denominator}} = \frac{\text{Product of Numerators}}{\text{Product of Denominators}}$$

For example, when multiplying **½ x ¾** , multiply the numerators, then the denominators and reduce or simplify the fraction if necessary.

½ x ¾ =

1 x 3 = 3 (Product of Numerators)

2 x 4 = 8 (Product of Denominators)

So the final product is **3/8**.It cannot be simplified (reduced) because the only common factor of 3 and 8 is 1.

If you are **multiplying mixed numbers**, simply convert the mixed numbers into improper fractions, multiply the fractions, and simplify the product if necessary.

For example, when multiplying **4 ¾ x 3 1/6** convert the mixed numbers into improper fractions (remember there is no need to find the common denominator since we are multiplying and not adding or subtracting).
4 ¾ = 19/4 (to convert the mixed number to an improper fraction, multiply the denominator 4 by the whole number 4, add the numerator 3 to obtain 19 and use the same denominator 4). **3 1/6 = 19/6** (6x3=18 +1= 19/6)

Therefore **19/4 x 19/6 = 361/24**.
(Numerator x numerator 19 x 19 = 361 and denominator x denominator 4 x 6 = 24). To convert this improper fraction
Into a mixed number, divide 361 by 24 for a result of 15 with a remainder of 1. Therefore, **15 1/24** is the final simplified product.

Try the next two examples on your own and check your answer below.

Example 1: Multiply $^{4}/_{7} \times {}^{14}/_{15}$.

Multiply 4 x 14 = 56 (product of numerators)

Multiply 7 x 15 = 105 (product of denominators)

Because 56 and 105 have factors in common, the product of 56/105 must be simplified.

Find the Greatest Common Factor (GCF) and divide both numerator and denominator by it. The GCF of 56 and 105 is **7**

(the factors of 56 are 1, 2, 4, **7**, 8, 14, 28, and 56 and the factors of 105 are 1, 3, 5, **7**, 15, 21, 35, and 105) so we divide both 56 and 105 by **7** to simplify the fraction.

$$\frac{56 \div 7}{105 \div 7} = \frac{8}{15}$$

The final, simplified product is $^{8}/_{15}$.

Example 2: Multiply $2\ ^{2}/_{3} \times 3\ ½$.

First, convert the mixed numbers into improper fractions; then multiply the fractions as you did above. **2 $^{2}/_{3}$ = 8/3** and **3 ½ = 7/2**.

Multiply **8/3 x 7/2 = 56/6**.

You must simplify **ANY** improper fraction by converting it into a mixed number. 56 divided by 6 gives a result of 9 with a remainder of 2. Therefore, **9 $^{2}/_{6}$** is the product However, 2 and 6 have some factors that are the same ("common" factors). Therefore, 2/6 MUST be simplified further.

To simplify 2/6, find the Greatest Common Factor (GCF) of 2 and 6, which is 2, and divide both 2 and 6 by 2 to obtain 1/3.

The final simplified product is **9 $^{1}/_{3}$**.

Remember, when dividing two numbers, like 6 **by** 4 or 6 ÷ 4, the first number is the **dividend**, the second number is the **divisor** and the answer is the **quotient**. When **dividing fractions**, replace the divisor by its **reciprocal** and then multiply to get your answer. To find the **reciprocal** of a number, you simply switch the numerator and denominator. For example, the reciprocal of **2/3 is 3/2**. If you wanted to find the reciprocal of 4, just think of 4 as **4/1** and then switch the numerator and denominator. You would get ¼.

It is also important to note that a fraction multiplied by its reciprocal ALWAYS gives a product of 1.

$$\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$$

Let's see an example of dividing fractions.

For example, divide ¾ by ½

The first step is to replace the divisor by its reciprocal and then multiply the fractions. **¾ x $^2/_1$ = $^6/_4$ = 1 $^2/_4$ = 1 ½**

The improper fraction $^6/_4$ was simplified to a mixed number **1 $^2/_4$** and then reduced to its lowest terms **1 ½.**

Let's take a look at **dividing mixed numbers**. To divide mixed numbers, first convert the mixed numbers to improper fraction (as when multiplying), then replace the divisor by its reciprocal and finally multiply the fractions.

For example, **3 $^1/_5$ ÷ 1 ¾**

First, convert each of the mixed numbers into improper fractions.

3 $^1/_5$ = 16/5 and **1 ¾ = 7/4.**

Then, replace the divisor by its reciprocal **7/4** becomes **4/7**

Finally, multiply the fractions. **16/5 x 4/7 = 64/35**

You MUST simplify the improper fraction. Divide 64 by 35 and the result is 1 with a remainder of 29. Therefore, the quotient is 1 $^{29}/_{35}$. 29 is a PRIME number. The ONLY factor that 29 and 35 have in common is the number 1. Therefore 29/35 can NOT be simplified (reduced).

The final quotient is **1 $^{29}/_{35}$**

Try the next examples on your own and check your answer.

Example 3: Divide **$^{1}/_{3}$ ÷ 2** Replace the divisor by its reciprocal.

The reciprocal of 2 or 2/1 is ½. Now we multiply **$^{1}/_{3}$ x ½ = $^{1}/_{6}$**.

This is the final quotient since **$^{1}/_{6}$** is in its lowest terms.

Example 4: Divide **$^{2}/_{9}$ ÷ 2 $^{2}/_{3}$**

Begin by converting the mixed number into an improper fraction

2 $^{2}/_{3}$ = $^{8}/_{3}$.

Next, replace the divisor by its reciprocal

8/3 becomes **3/8**

Then **multiply**. **$^{2}/_{9}$ x $^{3}/_{8}$ = $^{6}/_{72}$**.

6 and 72 have factors in common. The GCF is 6.

Therefore you MUST simplify **$^{6}/_{72}$**.

Dividing both 6 and 72 by 6, the quotient becomes **$^{1}/_{12}$**.

Therefore, the final (simplified) quotient is **$^{1}/_{12}$** .

Practice

Multiply or divide the following fractions and mixed numbers.

1. 2/5 **x** 2/3

a. $^{2}/_{15}$ b. $^{4}/_{15}$ c. $^{3}/_{5}$ d. 1 $^{1}/_{15}$

2. 5 x 2 ¼

a. $^{20}/_{9}$ b. 2 $^{2}/_{9}$ c. 10 $^{1}/_{4}$ d. 11 ¼

3. 4/9 ÷ 7

a. $^{28}/_{9}$ b. 3 $^{1}/_{9}$ c. $^{4}/_{63}$ d. $^{63}/_{4}$

4. 3 $^{1}/_{6}$ ÷ 3 $^{1}/_{3}$

a. $^{19}/_{20}$ b. 10 $^{5}/_{9}$ c. 10 $^{9}/_{5}$ d. $^{20}/_{19}$

5. Tiffany has a string that is 2 ½ feet long. If she cuts the string into equal pieces and each piece is ¼ foot long, how many pieces does she have?

a. $^{1}/_{10}$ b. $^{2}/_{20}$ c. $^{5}/_{8}$ d. 10

Answers to practice:

1. **b.** $^{4}/_{15}$
2. **d.** 11 $^{1}/_{4}$
3. **c.** $^{4}/_{63}$
4. **a.** $^{19}/_{20}$
5. **d.** 10

LESSON 4 THINGS TO REMEMBER

Comparing and Ordering Fractions

❖ When comparing and ordering fractions, you will use the same symbols (learned in Lesson 1) of greater than (>), less than (<) and equal to (=). You need to compare and order fractions with the same denominator, or with different denominators.

❖ **Remember:** A fraction is made up of a numerator and a denominator as shown below.

$$\frac{\textbf{Numerator}}{\textbf{Denominator}}$$

❖ **Comparing Fractions when the denominators are the same**

This is quite simple since all you have to do is look at the numerators of the fractions and compare those.

For example, when comparing $\frac{1}{4}$ and $\frac{3}{4}$, since the denominators are both 4, we are going to compare the numerators 1 and 3. Since 1 is less than 3, $\frac{1}{4}$ is less than $\frac{3}{4}$. Therefore, $\frac{1}{4} < \frac{3}{4}$.

❖ **Comparing Fractions when the denominators are different**

There are several ways this can be done.

❖ **Method 1:** Write equivalent fractions with a common denominator. Let's see how this is done. You need to use the concept of finding the Least Common Multiple of the denominators or the Least Common Denominator.

For example, when comparing $^5/_6$ and $^7/_8$, since the fractions have different denominators, we are going to write fractions equivalent to $^5/_6$ and to $^7/_8$.
We do this by finding the Least Common Multiple of the denominators or **LCD** (Least Common Denominator). The LCD for 6 and 8 is **24**.
(We listed the multiples for 6: 6, 12, 18, **24**, 30, 36, 42, 48…
and the multiples for 8: 8, 16, **24**, 32, 40, 48…).
Now we are going to create equivalent fractions.

$\frac{5}{6} = \frac{20}{24}$ Since 6 x 4 = 24, we need to multiply 5 x 4 = 20, to obtain an equivalent fraction.

Now we will do the same with $^7/_8$.

$\frac{7}{8} = \frac{21}{24}$ Since 8 x 3 = 24, we need to multiply 7 x 3 = 21, to obtain an equivalent fraction.

So now we can compare the new fractions $\mathbf{\frac{20}{24}}$ and $\mathbf{\frac{21}{24}}$.

We compare these two fractions as we did in the previous example.
Since, **20**< **21**, then **20/24**< **21/24** and **5/6** < **7/8**.
(Notice that, when we created the equivalent fractions,
5/6 became 20/24 and 7/8 became 21/24.)

❖ **Method 2:** A second way to compare fractions with unlike denominators is to convert the fractions into decimals and compare them as we did in Lesson 1.
Let's take a look at the same example using this method.

Let's begin by converting 5/6 and 7/8 into decimals.
5/6 = 0.8333 and **7/8 = 0.875**, since **0.875 is larger**, then **5/6** < **7/8**.

- **Method 3:** Another way this can be done is by cross-multiplying the products. Let's see.

We are going to cross-multiply the denominator of one fraction with the numerator of the other and vice versa.

$$\frac{5}{6} \times \frac{7}{8}$$

8 x 5 = 40 and 6 x 7 = 42, since 40 < 42, then 5/6 < 7/8.

- You may choose to compare and order fractions in any of the three methods shown above. It is especially important that you recall how to compare fractions using Method 1 since you will be using it when adding and subtracting fractions with unlike denominators in the next lesson.
- When comparing mixed numbers that have different whole numbers, simply compare the whole numbers. **If the whole numbers are the same, use the following example..**

For example, compare **3 $^{3}/_{4}$** and **3 $^{5}/_{16}$**.

Using Method 1, we compare $^{3}/_{4}$ and $^{5}/_{16}$ by making equivalent fractions, since the denominators are different. (Notice we don't discuss the whole number 3 since it is the same for both mixed numbers.)

We do this by finding the Least Common Multiple of the denominators or LCD (Least Common Denominator). The LCD for 4 and 16 is **16**.

(We listed the multiples for 4: 4, 8, 12, **16**, 20, 24, 28, 32, 36, 40, 44, 48 and the multiples for 16: **16**, 32, 48, 64, 80 …).

Now we are going to create fractions that are <u>equivalent to</u> $^{3}/_{4}$. and to $^{5}/_{16}$

$$\frac{3}{4} = \frac{12}{16}$$

Since 4 x 4 = 16, we need to multiply 3 x 4 = 12, to obtain a fraction that is equivalent to ¾. .Now we compare this to the $^{5}/_{16}$.

Since, 12/16 > 5/16, then **3/4** > **5/16**. Therefore, $3^{3}/_{4} > 3^{5}/_{16}$.

Using Method 2, we convert $\mathbf{3^{3}/_{4}}$ and $\mathbf{3^{5}/_{16}}$ into decimals and then compare.

$\mathbf{3^{3}/_{4} = 3.75}$ and $\mathbf{3^{5}/_{16} = 3.3125}$. Since **3.75** > **3.3125**, $\mathbf{3^{3}/_{4} > 3^{5}/_{16}}$.

Using Method 3, we can cross-multiply the denominator of one fraction with the numerator of the other and vice versa.

3   5

4   16

16 x 3 = 48 and 4 x 5 = 20, since **48** > **20**, then **3/4** > **5/16**.

Therefore, $\mathbf{3^{3}/_{4} > 3^{5}/_{16}}$.

❖ Try some examples of comparing fractions and mixed numbers on your own and check your answers below..

Example 1: Compare **8/11** and **5/7**.

Using Method 2, we convert 8/11 and 5/7 into decimals and then compare. **8/11 = 0.7272**… and

5/7 = 0.71429.

Since **0.7272 is greater than 0.71429**, **8/11** > **5/7**.

Example 2: Compare $\mathbf{6^{2}/_{7}}$ **and** $\mathbf{6^{1}/_{9}}$.

Using Method 3, we can cross-multiply the denominator of one fraction with the numerator of the other and vice versa.

2   1

7   9

9 x 2 = 18 and 7 x 1 = 7, since 18 > 7, then **2/7** >**1/9**.

Therefore, $\mathbf{6^{2}/_{7} > 6^{1}/_{9}}$.

We compare two fractions or mixed numbers at a time, and then we compare the third, fourth, and so on. Try the next example on your own and check your answer below.

Example 3: Order ½, $^3/_8$, $^5/_8$, and ¼ from least to greatest.

By looking at the fractions, we know that $^3/_8 < {}^5/_8$ (because 3 < 5 and the denominators are the same).

Now all we have to do is find out how ½ and ¼ compare to these.

Using Method 2, ½ > $^3/_8$ but **less than** $^5/_8$ because ½ = 0.5, **3/8 = 0.375, and 5/8 = 0.625**. If we were to order these from least to greatest ½ would be between $^3/_8$ and $^5/_8$ or $^3/_8 <$ ½ $< {}^5/_8$. Now we still need to compare ¼ to the rest of the fractions. ¼ **= 0.25** so it is less than all of the fractions.

Therefore, ¼ $< {}^3/_8 <$ ½ $< {}^5/_8$.

Adding and Subtracting Fractions

- When **adding or subtracting fractions with the same denominators**, you simply add or subtract the numerators and keep the denominators the same.

For example, 1/5 + 2/5.
Add the numerators 1 + 2 = 3, so 1/5 + 2/5 = 3/5.
Note that, in some cases, it may be necessary to simplify the answer.

- When **adding or subtracting fractions with different denominators**, you must first change each fraction to an equivalent fraction with the same denominator (as seen in the previous lesson, Method 1) and then you can find the sum (+) or difference (-) as seen in the previous example.

For example, 2/3 + 7/8.

Using Method 1, we find the Least Common Multiple of the Denominators or LCD. The LCD for 3 and 8 is **24**.

(We listed the multiples for 3: 3, 6, 9, 12, 15, 18, 21, **24**, 27, 30 …
and the multiples for 8: 8, 16, **24**, 32 …).

Now we are going to create equivalent fractions.

$\frac{2}{3} = \frac{16}{24}$ Since 3 x 8 = 24, we need to multiply the 2 by 8 also, to obtain a fraction that is equivalent to 2/3.

Now we will do the same with 7/8.

$\frac{7}{8} = \frac{21}{24}$ Since 8 x 3 = 24, we need to multiply the 7 by 3 also, to obtain a fraction that is equivalent to 7/8.

Now we can add the equivalent fractions with the same denominator.

16/24 + 21/24 = 37/24

37/24 is an improper fraction and needs to written as a mixed number, so we divide 37 by 24 and get a result of 1 with a remainder of 13.

Therefore, 2/3 + 7/8 = 1 $^{13}/_{24}$.

(Note that the whole number one is the result, the numerator 13 is the remainder, and 24 is the same denominator.)

Now we must simplify: 1 $^{13}/_{24}$, we simplify the fraction by finding the Greatest Common Factor of 13 and 24. This factor is 1 (13 is a prime number; factors are 1 and 13). This means that we can divide both the numerator and denominator by 1, leaving the fraction to be $^{13}/_{24}$

Therefore 2/3 + 7/8 = 1 $^{13}/_{24}$.

- **Adding and subtracting mixed numbers with the same and different denominators** is the same as adding and subtracting fractions with the same and different denominators. Keep in mind that you must add or subtract the whole numbers in the mixed numbers as well as the fractions themselves, and sometimes you will need to simplify an improper fraction in you answer as in the previous example.

Try the following examples on your own and check your answers below.

Example 1: Subtract 5/8 – 1/6.

Before subtracting the numerators, we must create equivalent fractions so that the denominators are the same. Refer to Method 1.to create equivalent fractions.

5/8 = 15/24 Since 8 x 3 = 24, we multiply the 5 by the 3 also, to obtain a fraction equivalent to 5/8.

Doing the same for 1/6: 1/6 = 4/24 Since 6 x 4 = 24, we multiply the 1 by the 4 also, to obtain a fraction equivalent to 1/6

Now we can subtract 15/24 – 4/24. 15 – 4 = 11,

so our answer is 11/24.

(Since 11 and 24 do not have a common factor other than 1, 11/24 is in its lowest terms, cannot be simplified, and therefore is the final answer.)

Example 2: Add **23 $^{7}/_{10}$ + 37 $^{3}/_{10}$**.

To add mixed numbers with the same denominator, you just need to place the sum of the numerators over the common denominator. Then add the whole numbers.

7/10 + 3/10 = 10/10 = 1

23 + 37 = 60 + 1 = 61

(Since the sum of the fractions was 10/10 = 1, we do not have a fraction in our answer. Notice how we simply added the sum of the fractions to the sum of the whole numbers to obtain our final answer).

Example 3: Add **11 $^{2}/_{3}$ + 10 $^{4}/_{5}$**.

Let's begin by creating equivalent fractions for 2/3 and 4/5 since the denominators are different.

2/3 = **10/15**

(we chose 15 as the LCD because it is the LCM of 3 and 5).

Since 3 x 5 = 15, then multiply 2 by the 5 = 10.

4/5 = **12/15**. Since 5 x 3 = 15, then multiply the 4 by 3 = 12.

Now we can add the new fractions

10/15 + **12/15** = **22/15**.

This is an improper fraction; therefore, it needs to be simplified. We divide 22 by 15 and the result is 1 with a remainder of 7. Therefore, 22/15 = **1 7/15**. (Remember how this was calculated earlier: the 15 is the same denominator of the equivalent fractions. And 7 and 15 do not have any factors in common, except 1, so 7/15 cannot be simplified)

Now we add the <u>sum of</u> the mixed numbers 11 + 10 = **21** to the <u>sum of</u> the fractions **1 $^{7}/_{15}$** + **21** = 22 $^{7}/_{15}$.

And **22 $^{7}/_{15}$** is the final answer.

- The last topic to discuss in this section is **subtraction of mixed numbers when you need to borrow or have to rename the mixed number**. Let's look at an example.

Subtract **$9\ ^{1}/_{5} – 4\ ^{4}/_{5}$**.

In this example, you don't have to find the common denominator because they are the same, but you cannot subtract 4 from 1 so you have to borrow from the 9 or rename. Recall that **$9 = 8\ ^{5}/_{5}$** because $^{5}/_{5}$ = 1 + 8 = 9. The reason you choose $^{5}/_{5}$ instead of any other fraction is because the common denominator of the two fractions is 5 and $^{5}/_{5}$ = 1. If the common denominator had been 23, you would choose **23/23** because **23/23** = 1. Now add the 8 $^{5}/_{5}$ to the $^{1}/_{5}$ of the 9 $^{1}/_{5}$ to rename the fraction. **$^{5}/_{5} + {}^{1}/_{5} = {}^{6}/_{5}$**

So **9 1/5** becomes **$8\ ^{6}/_{5}$**.

Now you can subtract **$8\ ^{6}/_{5} – 4\ ^{4}/_{5}$**

Subtract the fractions first **$^{6}/_{5} - {}^{4}/_{5} = {}^{2}/_{5}$**

and then the whole numbers **8 – 4 = 4**

So your final answer is **$4\ ^{2}/_{5}$**.

Example 4: Subtract **12 – 8 ¾.**

In this case you have a mixed number subtracted from a whole number. Notice that it would be WRONG to subtract 12 – 8 = 4 ¾. WHY?? BECAUSE, AFTER you take 8 (pies) away from 12 (pies), leaving 4 (pies), you STILL need to take away an additional **¾** (of a pie). Four whole pies will NOT be left in the kitchen, and CERTAINLY NOT 4 pies PLUS **¾** of another pie.

Rename 12 as 11 $^{4}/_{4}$ because $^{4}/_{4}$ = 1 + 11 = 12

Now subtract **$11\ ^{4}/_{4}$ – 8 ¾.** Begin by subtracting the fractions **$^{4}/_{4}$ – ¾ = ¼.** Now subtract the whole numbers **11 – 8 = 3**,

so our final answer is **3 ¼**

❖ **Multiplying and Dividing Fractions**

When **multiplying fractions** and dividing fractions, you do not need to find the common denominator as you did in addition and subtraction. All you need to do is multiply numerator by numerator and denominator by denominator, then simplify the fraction if necessary.

$$\frac{\text{Numerator}}{\text{Denominator}} \times \frac{\text{Numerator}}{\text{Denominator}} = \frac{\text{Product of Numerators}}{\text{Product of Denominators}}$$

For example, when multiplying **½ x ¾**, multiply the numerators, then the denominators and reduce or simplify the fraction if necessary.

½ x ¾ =

$$\frac{1 \times 3}{2 \times 4} = \frac{3}{8} \quad \begin{matrix}\text{(Product of Numerators)} \\ \text{(Product of Denominators)}\end{matrix}$$

So the final product is **3/8**.It cannot be simplified (reduced) because the only common factor of 3 and 8 is 1

❖ If you are **multiplying mixed numbers**, simply convert the mixed numbers into improper fractions, multiply the fractions, and simplify the product if necessary.

For example, when multiplying **4 ¾ x 3 1/6** convert the mixed numbers into improper fractions (remember there is no need to find the common denominator since we are multiplying and not adding or subtracting).

4 ¾ = 19/4 (to convert the mixed number to an improper fraction, multiply the denominator 4 by the whole number 4, add the numerator 3 to obtain 19 and use the same denominator 4).

3 1/6 = 19/6 (6x3=18 +1= 19/6)

Therefore **19/4 x 19/6 = 361/24**

(Numerator x numerator 19 x 19 = 361 and denominator x denominator 4 x 6 = 24).

To convert this improper fraction into a mixed number, divide 361 by 24 for a result of 15 with a remainder of 1. Therefore, **15 1/24** is the final simplified product.

Example 1: Multiply **$^4/_7$ x $^{14}/_{15}$**.

Multiply 4 x 14 = 56 (product of numerators)

Multiply 7 x 15 = 105 (product of denominators)

Because 56 and 105 have factors in common, the product of 56/105 must be simplified. Find the Greatest Common Factor (GCF) and divide both numerator and denominator by it. The GCF of 56 and 105 is **7**

(the factors of 56 are 1, 2, 4, **7**, 8, 14, 28, and 56 and the factors of 105 are 1, 3, 5, **7**, 15, 21, 35, and 105) so we divide both 56 and 105 by **7** to simplify the fraction.

56 ÷ 7 = 8

05 ÷ 7 = 15

The final, simplified product is **$^8/_{15}$**.

Example 2: Multiply **2 $^2/_3$ x 3 ½.**

First, convert the mixed numbers into improper fractions; then multiply the fractions as you did above.

2 $^2/_3$ = 8/3 and **3 ½ = 7/2**.

Multiply **8/3 x 7/2 = 56/6**.

You must simplify **ANY** improper fraction by converting it into a mixed number. 56 divided by 6 gives a result of 9 with a remainder of 2.

Therefore, **9 $^2/_6$** is the product. However, 2 and 6 have some factors that are the same ("common" factors). Therefore, 2/6 MUST be simplified further.

To simplify 2/6, find the Greatest Common Factor (GCF) of 2 and 6, which is 2, and divide both 2 and 6 by 2 to obtain 1/3.

The final simplified product is **9 $^1/_3$**.

- Remember, when dividing two numbers, like 6 **by** 4 or 6 ÷ 4, the first number is the **dividend**, the second number is the **divisor** and the answer is the **quotient**. When **dividing fractions**, replace the divisor by its **reciprocal** and then multiply to get your answer. To find the **reciprocal** of a number, you simply switch the numerator and denominator. For example, the reciprocal of **2/3 is 3/2**. If you wanted to find the reciprocal of 4, just think of 4 as **4/1** and then switch the numerator and denominator. You would get ¼.
- It is also important to note that a fraction multiplied by its reciprocal ALWAYS gives a product of 1 $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$

Let's see an example of dividing fractions.

For example, divide ¾ by ½

The first step is to replace the divisor by its reciprocal and then multiply the fractions. **¾ x $^2/_1$ = $^6/_4$ = 1 $^2/_4$ = 1 ½**

The improper fraction $^6/_4$ was simplified to a mixed number **1 $^2/_4$** and then reduced to its lowest terms **1 ½.**

- Let's take a look at **dividing mixed numbers**. To divide mixed numbers, first convert the mixed numbers to improper fraction (as when multiplying), then replace the divisor by its reciprocal and finally multiply the fractions.

For example, **$3\ ^1/_5 \div 1\ ^3/_4$**

First, convert each of the mixed numbers into improper fractions.

$3\ ^1/_5$ = 16/5 and **$1\ ^3/_4$ = 7/4.**

Then, replace the divisor by its reciprocal **7/4** becomes **4/7**

Finally, multiply the fractions. **16/5 x 4/7 = 64/35**

You MUST simplify the improper fraction. Divide 64 by 35 and the result is 1 with a remainder of 29. Therefore, the quotient is $1\ ^{29}/_{35}$. 29 is a PRIME number. The ONLY factor that 29 and 35 have in common is the number 1. Therefore 29/35 can NOT be simplified (reduced).

The final quotient is **$1\ ^{29}/_{35}$**

Lesson 5: Equivalent Forms of Numbers

In this lesson, you learn how to convert fractions, decimals, and percents into equivalent forms of numbers.

In addition, you learn about exponents, their meaning, and how to simplify numbers that contain exponents.

Lesson 5a: Fractions, Decimals, and Percents

Fractions, decimals, and percents can all be expressed in equivalent forms. For example: 1/5 is equivalent to 0.2 and to 20%. This is because percents describe a ratio out of 100 which can be simplified into other fractions. Let's take the example noted above: 20%.

Converting a Percent to a Fraction

20% is the equivalent to saying 20 out of 100 or 20/100. When that is reduced, the Greatest Common Factor in 20 and 100 is 20.
So the result is 1/5.

$$\frac{20\ (\div 20)}{100\ (\div 20)} = \frac{1}{5}$$

By completing this process, we have converted a percent to a fraction written in lowest terms.

Converting a Fraction to a Percent

Likewise, we can convert a fraction into a percent by doing the reverse. Take the fraction noted above (1/5) and make a proportion equivalent to 100 (since percentages are out of 100).

$$\frac{1}{5} = \frac{?}{100}$$

You can cross multiply and divide to solve the proportion to find the missing number, which is the percentage out of 100. Let's see.

1 x 100 = 100 **5 times ? = 100** **? = 100 ÷ 5 = 20 or 20%**

Converting a Percent to a Decimal

In order to convert a percent to a decimal, divide the percent by 100 (since percents are out of 100). **20% = 20/100 = 20 ÷ 100 = 0.20 or 0.2**

Converting a Decimal to a Percent

In order to convert a decimal to a percent, we do the opposite, which is to multiply the decimal by 100. **0.2 x 100 = 20 or 20%**

Converting a Decimal to a Fraction

To convert a decimal to a fraction, write the decimal as a number over some multiple of 10, then reduce it to its lowest terms.

0.2 is the equivalent to saying 2 tenths or 2/10 = 1/5

(once reduced to its lowest terms)

0.02 is the equivalent of saying 2 hundredths or 2/100 = 1/50

Converting a Fraction to a Decimal

To convert a fraction to a decimal, divide the numerator by the denominator. **1/5 = 1 ÷ 5 = 0.2**

Try some examples on your own and check your answers.

Example 1: Convert 4/5 into a decimal.

To convert 4/5 to a decimal, **divide 4 ÷ 5 = 0.8.**

Example 2: Convert 0.29 into a percent.

To convert 0.29 to a percent, multiply by 100**.**

0.29 x 100 = 29%.

Example 3: Convert 76 % into a fraction.

To convert 76% to a fraction place the percent over 100 or 76/100 and simplify the fraction.

76/100 = 19/25 (divide both numerator and denominator by the GCF: 4).

Practice

1. Which of the following decimals is equivalent to ¾?

   a. 0.3 b. 0.34 c. 0.75 d. 0.7

2. Which of the following percents is equivalent to $^6/_7$?

   a. 60% b. 70% c. 85.7% d. 66.7%

3. Which of the following fractions is equivalent to 0.4?

   a. 4/100 b. 2/5 c. 4/1000 d. 4/40

4. Which of the following decimals is equivalent to 45%?

   a. 0.45 b.0.045 c. 0.0045 d. 0.00045

5. Wal-Mart is advertising $100 bicycles for 33% off. Target is advertising the same bicycles at 1/3 off. Which is the better buy?

   a. Wal-Mart b. Target c. Neither d. They are the same

Answers to practice

1. c. 0.75
2. c. 85.7%
3. b. 2/5
4. a. 0.45
5. b. Target = 2/3 of $100 = $ 66.67. Wal-Mart = 67% of $100 = $67.00

Lesson 5b: Numbers with Exponents

Exponents are used to show a shortcut for repeated multiplication. For example, if you have 4 x 4 x 4 x 4 x 4 x 4, you write $\mathbf{4^6}$. The 4, the factor being multiplied, is called the **base**. The 6 is called the **exponent**, which tells us how many times to multiply the base. $\mathbf{4^6}$ is read as 4 to the sixth power. You'll notice that the exponent is printed smaller than, and is written slightly higher than, the base.

When writing the repeated multiplication of a number, make sure that it is the <u>same</u> factor. Count the amount of times the number is being multiplied, and then write the repeated multiplication using an exponent.

For example, 3 x 3 x 3 x 3 x 3 = $\mathbf{3^5}$. When evaluating $\mathbf{3^5}$, the answer is 243.

Evaluating the **square of a number** means to apply an exponent of 2 to the base.

For example, the square of 7 is the equivalent to having 7 as the base and 2 as the exponent (read as seven to the second power or seven squared) or $\mathbf{7^2}$ = 7 x 7 = 49.

Evaluating the **cube of a number** means to apply an exponent of 3 to the base.

For example, the cube of 6 is the equivalent to having 6 as the base and 3 as the exponent (read as six to the third power or six cubed) or $\mathbf{6^3}$ = 6 x 6 x 6 = 216.

Try some examples on your own and check your answer.

Example 1: Write the multiplication 9 x 9 x 9 x 9 x 9 using an exponent. Since all the factors are the same, count the number of times the 9 is being multiplied. The nine is being multiplied 5 times, so write $\mathbf{9^5}$ (read as nine to the fifth power), 9 being the base and 5 being the exponent.

Example 2: Write the multiplication of 8 x 8 x 8 x 2 x 2 using an exponent. In this multiplication, there are different factors, but you treat it the same way. To write this multiplication using exponents, count how many times 8 is being multiplied (3 times) and how many times 2 is being multiplied (2 times). Next, we write $\mathbf{8^3}$ x $\mathbf{2^2}$. This is read as “eight to the third power times two to the second power ” or ”eight cubed times two squared”.

Example 3: Evaluate $\mathbf{5^2}$.

$\mathbf{5^2}$ is equivalent to multiplying 5 (the base) 2 times or 5 x 5 = 25.

(A common mistake is to do this as 5 x 2 = 10, which is incorrect).

Example 4: Evaluate 10 cubed.

10 cubed is the same as $\mathbf{10^3}$ or 10 x 10 x 10 = 1000.

Example 5: Evaluate $\mathbf{7^4}$.

Seven to the fourth power or 7 x 7 x 7 x 7 = 2,401.

Practice

Write each multiplication using exponents.

1. a x a x a x b x b x b

a. a^3b^3 b. 3a x 3b c. 3ab d. a x 3 x b x 3

2. 1 x 1 x 1 x 1 x 1 x 1 x 1

a. 7 b. 1^7 c. 1^6 d. 6

Evaluate each power.

3. 6^4

a. 24 b. 216 c. 1,296 d. 6 x 6 x 6 x 6

4. 2^{12}

a. 4,096 b. 24 c.14 d. 2x2x2x2x2x2x2x2x2x2x2x2

5. 10^8

a. 80 b. 18 c. 10,000,000 d. 100,000,000

Answers to practice

1. **a.** a^3b^3
2. **b.** 1^7
3. **c.** 1,296
4. **a.** 4,096
5. **d.** 100,000,000

LESSON 5 THINGS TO REMEMBER

Numbers with Exponents

- Exponents are used to show a shortcut for repeated multiplication. For example, if you have 4 x 4 x 4 x 4 x 4 x 4, you write $\mathbf{4^6}$. The 4, the factor being multiplied, is called the **base**. The 6 is called the **exponent**, which tells us how many times to multiply the base. $\mathbf{4^6}$ is read as 4 to the sixth power. You'll notice that the exponent is printed smaller than, and is written slightly higher than, the base.
- When writing the repeated multiplication of a number, make sure that it is the same factor. Count the amount of times the number is being multiplied, and then write the repeated multiplication using an exponent.

For example, 3 x 3 x 3 x 3 x 3 = $\mathbf{3^5}$. When evaluating $\mathbf{3^5}$, the answer is 243.

Percent of a Number

For example, find 75% of 36.

First, we convert 75% to a decimal = 0.75 then we multiply by 36.

0.75 x 36 = 27 therefore, 75% of 36 is 27.

If we used the fraction equivalent of 75% = $\frac{75}{100} \times \frac{36}{1} = \frac{2700}{100} = 27$

What percent of 64 is 288? In this problem there is <u>no percent</u>, but there <u>is</u> the "of" or the <u>whole</u> and the "is" or the <u>part</u>. $\frac{\%}{100} = \frac{288}{64}$ (**Do not assume that the smaller number is the part or "is". It is not the case in this example**.) Now we cross-multiply and divide. 288 x 100 = 28,800 Divide by 64 = 450%.
28 is 40% of what number? In this problem, we have a <u>percent</u> and the <u>part</u> or "is". Let's set it up. $\frac{40}{100} = \frac{28}{\mathbf{x}}$ Now we cross-multiply 40 **x** = 28 x 100 = 2,800 Then divide **x** = 2800/40.= 70 Therefore 70 is the <u>whole</u> or "of".

<u>Fractions, Decimals, and Percents</u>

- Fractions, decimals, and percents can all be expressed in equivalent forms. For example: 1/5 is equivalent to 0.2 and to 20%. This is because percents describe a ratio out of 100 which can be simplified into other fractions.

Let's take the example noted above: 20%.

Converting a Percent to a Fraction

20% is the equivalent to saying 20 out of 100 or 20/100. When that is reduced, the Greatest Common Factor in 20 and 100 is 20.

So the result is 1/5.

$$\frac{20\ (\div 20)}{100\ (\div 20)} = \frac{1}{5}$$

By completing this process, we have converted a percent to a fraction written in lowest terms.

Converting a Fraction to a Percent

Likewise, we can convert a fraction into a percent by doing the reverse. Take the fraction noted above (1/5) and make a proportion equivalent to 100 (since percentages are out of 100).

$$\frac{1}{6} = \frac{?}{100}$$

You can cross multiply and divide to solve the proportion to find the missing number, which is the percentage out of 100. Let's see.

1 x 100 = 100 **5 times ? = 100** **? = 100 ÷ 5 = 20 or 20%**

Converting a Percent to a Decimal

In order to convert a percent to a decimal, divide the percent by 100 (since percents are out of 100). **20% = 20/100 = 20 ÷ 100 = 0.20 or 0.2**

Converting a Decimal to a Percent

In order to convert a decimal to a percent, we do the opposite, which is to multiply the decimal by 100. **0.2 x 100 = 20 or 20%**

Converting a Decimal to a Fraction

To convert a deci mal to a fraction, write the decimal as a number over some multiple of 10, then reduce it to its lowest terms.

0.2 is the equivalent to saying 2 tenths or 2/10 = 1/5

(once reduced to its lowest terms)

0.02 is the equivalent of saying 2 hundredths or 2/100 = 1/50

Converting a Fraction to a Decimal

To convert a fraction to a decimal, divide the numerator by the denominator. 1/5 = 1 ÷ 5 = 0.2

Example 1:	Convert 4/5 into a decimal. To convert 4/5 to a decimal, **divide 4 ÷ 5 = 0.8.**
Example 2:	Convert 0.29 into a percent. To convert 0.29 to a percent, multiply by 100**.** 0.29 x 100 = 29%.
Example 3:	Convert 76 % into a fraction. To convert 76% to a fraction place the percent over 100 or 76/100 and simplify the fraction. 76/100 = 19/25 (divide both numerator and denominator by the GCF: 4).

Lesson 6: Percents

In Lesson 5a you learned about percent and its equivalent forms. You learned how to change a percent to a fraction and a decimal. You learned also how to convert a decimal into a percent and a fraction and how to convert a fraction to a percent and a decimal. These tools will help you in this lesson.

Lesson 6a: Percent of a Number

Recall what you learned about percent and its equivalent forms. Use this information about how to change percents, fractions, and decimals to alternate forms in order to find the percent of a number.
How do you find the percent of a number? There are several ways. One way is to convert the percent to a fraction or decimal and then multiply by the number.

For example, find 75% of 36.

First, we convert 75% to a decimal = 0.75 then we multiply by 36.

0.75 x 36 = 27 therefore, 75% of 36 is 27.

If we used the fraction equivalent of 75% = $\frac{75}{100} \times \frac{36}{1} = \frac{2700}{100} = 27$.

An alternate way of finding the percent of a number is to use the **Proportion Method**.

The proportion is set up as: $\frac{\%}{100} = \frac{\text{is (part)}}{\text{of (whole)}}$

This formula is set up so that, regardless of what components you are given in a problem, you can set up the proportion and solve it the same way each time.

Always place the percentage in place of the %, the "is" is replaced by the part of the base or whole, and the "of" is replaced by the base amount or the whole. Notice that 100 is constant because percentages are always out of 100.

If you are given the value for ANY two of the three (**%**, **part**, **total**), then you can calculate the value of the third. In order to solve this proportion, substitute the values that are given in the problem, cross multiply and then divide.

For example: Find 65% of 200 using a proportion.

In this problem, simply convert the percentage to a decimal or fraction and then multiply by 200, but this problem requires the use a proportion.

What part of 200 is 65%? $\frac{65}{100} = \frac{\mathbf{x}}{200}$

We set up the proportion by replacing the % with 65 and the "of" or whole with 200. Now we can cross-multiply $100\,\mathbf{x} = 65 \times 200 = 13000$

Then divide both sides by 100. $\mathbf{x} = 13000 \div 100 = 130.$

Therefore, 130 is 65% of 200.

The **Proportion Method** is especially effective because we don't have to learn alternate ways to solve the percent of a number based on the information given. Let's see.

What percent of 64 is 288?

In this problem there is no percent, but there is the "of" or the whole and the "is" or the part.

$$\frac{\%}{100} = \frac{288}{64}$$

(**Do not assume that the smaller number is the part or "is". It is not the case in this example**.)

Now we cross-multiply and divide.

288 x 100 = 28,800 Divide by 64 = 450%.

28 is 40% of what number?

In this problem, we have a percent and the part or "is". Let's set it up.

$$\frac{40}{100} = \frac{28}{\mathbf{x}}$$

Now we cross-multiply 40 **x** = 28 x 100 = 2,800

Then divide **x** = 2800/40.= 70

Therefore 70 is the whole or "of".

So, you see, regardless of what is missing in the problem, the Proportion Method is useful in all three cases: when either the percent, the part or the whole is missing. Try the following examples on your own and check your answers.

Example 1: What is 33% of 68?

Let us begin by setting up the proportion.

$$\frac{33}{100} = \frac{\mathbf{x}}{68}$$

Cross-multiply 100 x **x** = 33 x 68 = 2,244

Then, divide both sides by 100 **x** = 2244/100 = 22.24.

Therefore, 22.44 is the <u>part</u> or "is".

Example 2: 63 is 90% of <u>what</u> number?

Set up the proportion.

$$\frac{90}{100} = \frac{63}{\mathbf{x}}$$

Cross-multiply 90 x **x** = 63 x 100 = 6,300

Then, divide both sides by 90 **x** = 6300/90 = 70

Therefore, 70 is the <u>whole</u> or "of".

Example 3: What percent of 55 is 33?

Set up the proportion.

$$\frac{\%}{100} = \frac{33}{55}$$

Cross-multiply 55 x **%** = 33 x 100 = 3,300

Then, divide both sides by 55 **%** = 3300/55 = 60

Therefore, 60% is the missing percent in the problem.

Practice

Solve the following percent problems.

1. 15 is 75% of what number?

   a. 500   b. 325   c. 20   d. 11.25

2. What percent of 240 is 80?

   a. $33\ ^{1}/_{3}\%$ b. 192%   c. 230%   d. 300%

3. 150% of 48 is what number?

   a. 32   b. 72   c. 257.5   d. 312.5

4. 21% of 36 is what number?

   a. 171.43   b. 112.79   c. 58.3   d. 7.56

5. The sales tax in the State of Florida is 7%. If we were to buy a lawnmower that costs $175, what would we have to pay in sales tax?

   a. $12.25   b. $122.50   c. $162.85   d. $187.25

Answers to practice

1. c. 20

2. a. $33^{1}/_{3}\%$

3. b. 72

4. d. 7.56

5. a. $12.25

Lesson 6b: Percent of Increase and Decrease

Percent of increase and decrease are useful when comparing monthly expenses, a company's gains (on a monthly, quarterly, or yearly basis), stocks, or just about any numerical value that has either increased or decreased throughout a period of time.

To find the percent of increase or decrease, continue to use the Proportion Method used in Lesson 6a. This time, though, the part or "is" is considered to be amount of increase or decrease and the whole or "of" is the original amount.

$$\frac{\textbf{\% of increase/decrease}}{\textbf{100}} = \frac{\textbf{amount of increase/decrease}}{\textbf{original amount}}$$

Let us find the percent of increase.

Find the percent of increase from 56 to 70.

The first step is to find out the amount of increase: 70 – 56 = 14

The second step is to establish the original amount: 56

The third step is to set up the proportion: $\frac{\%}{100} = \frac{14}{56}$

Then we cross-multiply 56 x **%** = 14 x 100 = 1400

Finally, divide both sides by 56 **%** = 1400/56 = 25.

Therefore, 25% is the percent of increase from 56 to 70.

Let us find the percent of decrease.

Find the percent of decrease from 225 to 189.

The first step is to find out the amount of decrease: 225 – 189 = 36

The second step is to establish the original amount: 225

The third step is to set up the proportion: $\frac{\%}{100} = \frac{36}{225}$

Then, cross-multiply 225 x **%** = 36 x 100 = 3600

Finally, divide both sides by 225 **%** = 3600/225 = 16.

Therefore, 16% is the percent of decrease from 225 to 189.

Try the following examples on your own and check your answer.

Example 1: Find the percent of increase from 20 to 39.

The amount of increase: 39 – 20 = 19

The original amount: 20

$\frac{\%}{100} = \frac{19}{20}$

Cross-multiply 20 x **%** = 19 x 100 = 1900

Finally, divide both sides by 20 **%** = 1900/20 = 95

Therefore, 95% is the percent of increase from 20 to 39.

Example 2: The price of a suit is reduced from $175 to $91. What is the percentage of decrease from the original price to the sale price? The amount of decrease: $175 – $91 = $84

The original amount: $175

$\frac{\%}{100} = \frac{\$84}{\$175}$

Cross-multiply $175 x **%** = 84 x 100 = 8400

Then, divide both sides by 175 **%** = 8400/175 = 48

Therefore, 48% is the percent of decrease from the original price to the sale price.

Practice

Find the percent of increase or decrease in the following problems.

1. 112 to 42

   a. 37.5% b. 62.5% c. 60% d. 166.7%

2. 45 to 99

   a. 45.54% b. 54.54% c. 120% d. 220%

3. 46 to 23

   a. 25% b. 50% c. 75% d. 150%

4. 120 to 84

   a. 30% b. 42.9% c. 70% d. 233.3%

5. Yearly sales at Bloomingdale's dropped from $85 million in 2006 to $66.3 million in 2007. What is the percent of decrease in sales?

   a. 20% b. 22% c. 28% d. 78%

Answers to practice

1. **b.** 62.5%
2. **c.** 120%
3. **b.** 50%
4. **a.** 30%
5. **b.** 22%

Lesson 6c: Discounts and Sale Prices

A **discount** is the amount that an item is reduced from its original price. The **sale price** is the original price minus the discount. You can use the **Proportion Method** to determine discount, sale prices, as well as original prices of items. Let's take a look.

If the regular price of an item is $813.25 and the discount is 20%, what is the dollar value of the discount and what is the sale price of the item?

Two amounts are being asked in this problem: (1) the amount of the discount, and (2) the sale price.

Let's begin by finding the amount of the discount.

Set up a proportion using 20% and $813.25 as the original price (the whole).

$$\frac{20}{100} = \frac{x}{813.25}$$

Cross-multiply 100 x **x** = 20 x $813.25 = 16,265

Then, divide by 100 = **$162.65, the amount of the discount** (the part).

Now we need to find the sale price. Recall from above that the sale price = original price – the discount.

Therefore, $813.25 - $162.65 = **$650.60 is the sale price**.

Note that some problems will ask only for the discount (only requiring the first step) and others will ask for the sale price, but not the discount. In this case, even though the problem may not say to find the discount, you must find it in order to obtain the sale price.

Find the discount and the sale price of an item that costs $1,200 and has a discount of 12%. In order to find the amount of the discount, use the Proportion Method. $\frac{12}{100} = \frac{x}{1200}$

($1,200 is the original price, therefore it is the whole or "of").

Cross-multiply 100 x **x** = 12 x $1,200 = $14,400

Then, divide both sides of the equation by 100 **x** = $14,400/100 = $144.

The **discount is $144**.

The sale price is: $1,200 (original) - $144 (discount) = **$1,056 (sale price).**

Try the next example on your own and check your answer below.

Example 1: A washing machine is on sale for 15% off the regular price of $689. What is the sale price of the washing machine? Even though this problem asks only for the sale price, you need to find the discount in order to find the sale price. You are going to use the Proportion Method.

Set up the proportion: $\frac{15}{100} = \frac{x}{689}$

Cross-multiply 100 x **x** = 15 x 689 = 10,335

Then, divide both sides by 100 **x** = 10335/100 = $103.35.

This is not the final answer because the problem asks for the sale price.

Therefore, $689 - $103.35 = $585.65 is the sale price of the washing machine.

Practice

Solve the following discount and sales price problems.

1. Find the discount of an item regularly priced at $17.89 with a discount of 10%.
   a. $1.79 b. $16.10 c. $17.89 d. $19.68

2. Find the regular price of an item if the discount amount is $45 (that is 18%).
   a. $8.10 b. $36.90 c. $40 d. $250

3. Find the sale price of an item that regularly costs $46 and has a 20% discount.
   a. $9.20 b. $36.80 c. $184 d. $230

4. If an item has a 30% discount and is priced at $18.90, what is the sale price?
   a. $5.67 b. $13.23 c. $44.10 d. $63

5. Marie needed a calculator for her mathematics class. She saw an add in the newspaper for Office Depot that announced all calculators having 15% discount. If the calculator Marie wanted was listed for $12.99, what was the sale price of the calculator?
   a. $1.95 b. $11.04 c. $73.61 d. $86.60

Answers to practice

1. a. $1.79

2. d. $250

3. b. $36.80

4. b. $13.23

5. b. $11.04

Lesson 6d: Simple Interest

When you have a bank account, usually savings, the bank pays you because it is lending your money to other customers. This amount that the bank pays you for the use of your money is called **interest**. Also, when you borrow money from the bank, a loan, the bank charges you higher interest for the use of its depositors' money. In both cases, there are three factors involved:

1) **interest rate** earned by your money or charged by the lender
2) **principal** which is the amount you have in the bank or are borrowing from the bank
3) duration of **time** your money stays in the bank or the time you take to repay the loan.

The formula for finding **simple interest** is:

$$I = P \times R \times T$$

I = represents the interest amount (dollars and cents)

P = represents the principal (dollars and cents)

R = represents the interest rate (percent per time period)

T = represents the number of time periods (days, weeks, months, years)

The interest rate's time period MUST BE the SAME AS the time's periods. If interest rate is 18% per <u>YEAR</u>, then Time MUST be calculated in <u>YEARS</u>.

If interest rate is 2.225% per <u>DAY</u>,
then Time MUST be calculated in <u>DAYS</u>.

Let's look at an example.

Dennis wants to borrow $5,000 from his bank. The current annual interest rate is 7% for 3 years. How much will he owe the bank after 3 years?

In this problem the principal = $5,000, rate = 7% per year, and time = 3 years.

So we apply the formula**:** ***I = P x R x T***

I = 5,000 x 0.07 x 3 = **$1,050 is the amount of the interest** alone that Dennis will have to pay back. Since we want to find out how much he will owe the bank in total, we add the principal $5,000 + the interest $1,050 = for a **total of $6,050 ,** assuming that he has made no payments on this loan..

(Notice that the percentage was converted to a decimal in order to multiply.)

Try the next example on your own and check your answer below.

Example 1: Denise is going to open a savings account. She wants to deposit $1,750 for a period of 2 ½ years.

The annual interest rate is 1.5%. How much interest will she have earned at the end of the 2 ½ years? How much money will she have at the end of this period?

The principal = $1,750, the annual interest rate = 1.5% or 0.015, and the time = 2.5 years. Now let us apply the formula.

I = P x R x T = 1,750 x 0.015 x 2.5 = **$65.63 is the amount of the interest** that Denise will have earned after 2.5 years.

To find out how much Denise will have in her savings account at the end of this period, **add** the principal $1,750 to the interest earned $65.63 for a total of **$1,815.63**.

Practice

Solve the following simple interest problems.

1. Which of the following is the interest earned on a principal of $4,800 and an annual interest rate of 12.5% for 3 years?

   a. $1,800 b. $6,600 c. $180,000 d. $184,800

2. The total amount owed at the termination of a loan for $2,500 at an annual interest rate of 3.5% for 1½ years is __________.

   a. $13.13 b. $131.25 c. $2,513.13 d. $2,631.25

3. Bobby borrowed $9,000 from his neighborhood bank to buy a used car. The bank gave him an annual interest rate of 7.5% for 2 ½ years. How much interest did he pay throughout the 2 ½ years period?

   a. $168.75 b. $1,687.50 c. $9,168.75 d. $10,687.50

4. Katherine opened a savings account at a rate of 9% per year. Her initial and only deposit was for $1,500. At the end of two years, how much will she have in her account?

   a. $270 b. $1,770 c. $27,000 d. $28,500

5. Todd and Joseph opened a savings account at different banks. They both deposited $1,000 to start. Todd's bank offers a 2% annual interest rate for a period of 3 ½ years. Joseph's bank offers an annual interest rate of 3.25% for 2 years. Which of the two banks will yield the greater interest at the end of the term?

   a. Todd's   b. Joseph's   c. Neither   d. They are the same.

Answers to practice

1. a. $1,800

2. d. $2,631.25

3. b. $1,687.50

4. b. $1,770

5. a. Todd's

LESSON 6 THINGS TO REMEMBER

❖ **Example 1**: (From p. 71 of the text) 28 is 40% of what number?

In this problem, we have a percent and the part or "is". Let's set it up.

$$\frac{40}{100} = \frac{28}{\mathbf{x}}$$

Now we cross-multiply 40 **x** = 28 x 100 = 2,800

Then divide **x** = 2800/40.= 70

Therefore 70 is the whole or "of".

❖ **Example 2**: find 75% of 36.

First, we convert 75% to a decimal = 0.75 then we multiply by 36.

0.75 x 36 = 27 therefore, 75% of 36 is 27.

If we used the fraction equivalent of 75% = $\frac{75}{100} \times \frac{36}{1} = \frac{2700}{100} = 27.$

❖ **Example 3:** 30 is what % of 90?

Some would say 30 is what part of 90?

The 30/90=3/9 = 1/3 or .333 or 33.3%

❖ **Example 4:** 65% of 940 students at a school failed at least one subject. How many student failed at least one subject? Change 65% to .65 and multiple .65 X 940. = 611.

❖ **Example 5:** A student earned 90% on a 160 question exam. How many points did he earn? Change 90% to .90 and multiply .90 X 160 = 144.

- **Example 6:** The sales tax in Florida is 7%. If you purchase a pair of shoes on sale for $40, what would your total bill including tax be" Change 7% to .77 and multiply .07 X $40 = $2.80. Now add $40 + $2.80 = $42.80.

- **Example 7:** The regular price for the new car you want is $20,000, and is on sale for 10% off. How much is the car? Change 10% is 10/100 or .10. Multiply, 10 X $20,000= $2000 and subtract $2,000 from $20,200= $18,000.

Percent of Increase and Decrease

- Percent of increase and decrease are useful when comparing monthly expenses, a company's gains (on a monthly, quarterly, or yearly basis), stocks, or just about any numerical value that has either increased or decreased throughout a period of time.

Find the percent of increase from 56 to 70.

The first step is to find out the amount of increase: 70 – 56 = 14

The second step is to establish the original amount: 56

The third step is to set up the proportion: $\frac{\%}{100} = \frac{14}{56}$

Then we cross-multiply 56 x **%** = 14 x 100 = 1400

Finally, divide both sides by 56 **%** = 1400/56 = 25.

Therefore, 25% is the percent of increase from 56 to 70.

Find the percent of decrease from 225 to 189.

The first step is to find out the amount of decrease: 225 – 189 = 36

The second step is to establish the original amount: 225

The third step is to set up the proportion: $\frac{\%}{100} = \frac{36}{225}$

Then, cross-multiply 225 x **%** = 36 x 100 = 3600

Finally, divide both sides by 225 **%** = 3600/225 = 16.

Therefore, 16% is the percent of decrease from 225 to 189.

Simple Interest

- The formula for finding **simple interest** is:

$$I = P \times R \times T$$

I = represents the interest amount (dollars and cents)

P = represents the principal (dollars and cents)

R = represents the interest rate (percent per time period)

T = represents the number of time periods (days, weeks, months, years)

The interest rate's time period MUST BE the SAME AS the time's periods. If interest rate is 18% per YEAR, then Time MUST be calculated in YEARS.

Example: Dennis wants to borrow $5,000 from his bank. The current annual interest rate is 7% for 3 years. How much will he owe the bank after 3 years?

In this problem the principal = $5,000, rate = 7% per year, and time = 3 years.

So we apply the formula**:** ***I = P x R x T***

I = 5,000 x 0.07 x 3 = **$1,050 is the amount of the interest** alone that Dennis will have to pay back. Since we want to find out how much he will owe the bank in total, we add the principal $5,000 + the interest $1,050 = for a **total of $6,050,** assuming that he has made no payments on this loan.

(Notice that the percentage was converted to a decimal in order to multiply.)

Lesson 7: Order of Operations

In this lesson, you learn how solving a problem with multiple operations can give different answers if you do not follow a specified set of rules. You will also learn these rules and how to apply them in a problem setting.
Solving problems might involve using more than one operation. Your answer could depend on the order in which you perform these operations. Often times, parentheses are used to determine which operations need to be done first.

For example, 10 + 5 x 3.

You can add 10 + 5 = 15 then multiply by 3 = 45

Or you could multiply 5 x 3 = 15 then add 10 = 25.

Which one of these would be correct?

To avoid confusion and to make sure everyone gets the same answer, mathematicians developed the following set of rules.

Order of Operations

Step 1: Simplify operations within parentheses.

Step 2: Simplify any exponents.

Step 3: Multiply and divide as it appears from left to right.

Step 4: Add and subtract as it appears from left to right.

Let's take a look at some examples.

Simplify 7 x (3 + 2).

You **MUST** simplify inside the parentheses FIRST. (3 + 2) = 5.
Now you have 7 x (5) = 35.

Simplify **$12 \times 3^2 + 7$.**

In this problem, you have no parentheses so begin with Step 2
Simplify the exponent $3^2 = 3 \times 3 = 9$.

Now you have 12 x 9 + 7. Since multiplication is done before addition,

(Step 3 before Step 4), multiply 12 x 9 = 108 **then** add 7 108 + 7 = **115**.

Try the following examples on your own and check your answers below.

Example 1: Simplify $(3^2 + 4)^3$.

Step 1 is simplify inside the parentheses. In order to do so, you have to simplify an exponent. That is Step 2. **$3^2 = 9$**.

Now add 4: 9 + 4 = (13).

Now continue Step 2 by evaluating **$(13)^3 = 2{,}197$**.

Example 2: Simplify 9 **x** 6 ÷ 2 – 3

In this problem you do not have any parentheses or exponents so proceed to Step 3, which is multiplication and division.

In this problem you have multiplication and division.

Since multiplication appears first, multiply 9 x 6 = 54.

Then divide by 2 = 27 and finally subtract 3 = 24.

Example 3: Simplify **$44 + 5 \times (4^2 \div 8)$**

Step 1: working inside the parentheses. Evaluate the exponent first: $4^2 = 4 \times 4 = 16$ Then divide 16 by 8 = 2.

Now you are left with 44 **+** 5 **x** (2). Do the multiplication first.

(Step 3) 5 x 2 = 10 and then (Step 4) add 10 **+** 44 = 54.

Practice

Simplify each expression below.

1. $(6^2 - 15 \div 3) \times 2^2$

a. 16 b. 124 c. 46 d. 1.75

2. $9 - (4 - 1)^2$

a. 0 b. 4 c. 6 d. 16

3. $40 + 18 \div 2 - 16$

a. 13 b. 33 c. 47 d. 63

4. $2 \times (6 - 5)^2$

a. 2 b. 13 c. 22 d. 38

5. The dance committee needs 5 balloons at each of 12 tables. They also need 45 balloons for each of the four walls of the room. For other decorations, they need 25 balloons, and the committee will order 8 extra balloons. Which of the following is the correct order of operations?

a. 5 + 12 + 45 + 4 + 25 + 8
b. 5 x 12 + 45 x 4 + 25 + 8
c. 5 x 12 + 45 x 4 + 25 x 8
d. 5 + 45 x 12 + 4 + 25 x 8

Answers to practice

1. **b. 124**
2. **a. 0**
3. **b. 33**
4. **a. 2**
5. **b. 5 x 12 + 45 x 4 + 25 + 8**

LESSON 7 THINGS TO REMEMBER

Order of Operations

Step 1: Simplify operations within parentheses.

Step 2: Simplify any exponents.

Step 3: Multiply and divide as it appears from left to right.

Step 4: Add and subtract as it appears from left to right.

Simplify 7 x (3 + 2).

You MUST simplify inside the parentheses FIRST. (3 + 2) = 5.

Now you have 7 x (5) = 35.

Simplify **12 x 3^2 + 7.**

In this problem, you have no parentheses so begin with Step 2

Simplify the exponent 3^2 = 3 x 3 = 9.

Now you have 12 x 9 + 7. Since multiplication is done before addition,

(Step 3 before Step 4), multiply 12 x 9 = 108

then add 7 108 + 7 = **115**.

Try the following examples on your own and check your answers below.

Example 1: Simplify $(3^2 + 4)^3$.

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Now add 4: 9 + 4 = (13).

Now continue Step 2 by evaluating **$(13)^3$ = 2,197**.

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Since multiplication appears first, multiply 9 x 6 = 54.

Then divide by 2 = 27 and finally subtract 3 = 24.

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Now you are left with 44 **+** 5 **x** (2). Do the multiplication first.

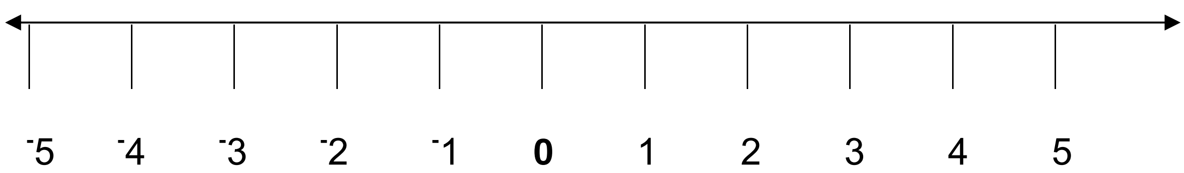
(Step 3) 5 x 2 = 10 and then (Step 4) add 10 **+** 44 = 54.

Lesson 8: Integers

In this lesson, you learn about positive and negative numbers and where we see them in the real world. You also learn about the opposite of a number as well as its absolute value. Finally, you use these facts about integers to perform operations such as addition, subtraction, multiplication, and division of integers.

Lesson 8a: Positive and Negative Numbers and Absolute Value

You are already familiar with positive numbers because they are the same as the Whole Numbers (0, 1, 2, 3, 4, …). **Integers** include this set of numbers **in addition to** including negative numbers. Let's take a look at the number line below to see the position and value of these numbers.



Integers are positive and negative numbers. Positive numbers are either shown with a plus ($^{+}$) sign or not. Negative numbers **always** have a negative ($^{-}$) sign to their left. This is a minus sign raised to the position of an exponent ("superscript"). The reason the negative numbers are always shown with a negative ($^{-}$) sign is to distinguish them from positive numbers. If a number does not have a sign, automatically assume it is positive.

It is important to note that 0 is neither positive nor negative.

On the number line above, the further the number is on the right side, the greater it is. The further to the left it is, the smaller it is.

For example, $^{-}5 < 0$ and $^{-}2 > {}^{-}10$.

The further away (to the left) a **negative** number is from 0 the **smaller** it is. Likewise, the further away (to the right) a **positive** number is from 0 the **greater** it is. Positive numbers are always greater than negative numbers for the same reason (they are further to the right).

Often integers are used to describe real-life situations. Take a look at some examples of how this is done.

For example, 10° below zero is expressed with an integer as $^{-}10$.
And 2° below zero is WARMER (greater temperature) than 10° below zero, as shown at the top of this page **$^{-}2 > {}^{-}10$**.
Another example, a gain of \$150 is expressed as $^{+}$ \$150 or simply \$150.
A loss of \$150 would be expressed as $^{-}$ **\$150**.
Another example, Jonathan lost 5 lbs. last week is represented as **$^{-}5$**.

Try the following examples on your own and check your answers.

Example 1: Which temperature is greater, **$^{-}7°$ or $^{-}6°$**?

Recall from above that the further the negative integer is away from (to the left of) 0, the smaller it is. In this case, **$^{-}7°$** is further from 0 than $^{-}6°$. 7° below zero is COLDER (smaller) than 6° below zero. Therefore, $^{-}7° < {}^{-}6°$.

Example 2: Order the following integers from least to greatest:

${}^{-}2$, ${}^{-}6$, 3, 7, ${}^{-}8$, 9, and ${}^{-}13$.

Since we know that the negative integers are smaller than the positive, let's order the negative from least to greatest.

The negative integers are**:** **${}^{-}2$, ${}^{-}6$, ${}^{-}8$, and ${}^{-}13$**.

The smallest of these is ${}^{-}13$, because it is the furthest from 0.

Then we have ${}^{-}8$, ${}^{-}6$, and ${}^{-}2$. Therefore, ${}^{-}13 < {}^{-}8 < {}^{-}6 < {}^{-}2$.

Now look at the positive integers: 3, 7, 9.

These are already in order from least to greatest; so**:**

$13 < {}^{-}8 < {}^{-}6 < {}^{-}2 < 3 < 7 < 9$.

Example 3: Write the following examples as an integer:

Joanne hopped 3 ft. backward = **${}^{-}3$**

Jeanie is 45 in. tall = **45**

Bob's stock went down 1 ¾ points = **${}^{-}1$ ¾**

Ann deposited \$50 in her account = **50**

Ann withdrew \$50 from her account= **${}^{-}50$**

Absolute value of a number is the distance the number is away from 0. Since absolute value refers to a distance, it can never be negative. You either travel or you don't, but **distance is never negative**.

Absolute value is denoted by these symbols $|\,|$**.** For example, $|5|$. This is read as "absolute value of 5". The absolute value of 5 = 5. Recall that if a number is inside these symbols it is asking, "how far is this number away from 0?" Keep in mind: **distance**.

For example, the absolute value of ${}^{-}4$ is written as $|{}^{-}4|$ = 4.

Absolute value should not be confused with the **opposite** of an integer. Every integer has an opposite and it has nothing do with distance, but it has to do with the opposite **value** of what it is.

For example,	The opposite of 4 is ¯4
	The opposite of 4º above zero is 4º below zero.
	The opposite of ¯4 is 4
	The opposite of 4º below zero is 4º above zero.
	The opposite of losing 5 lbs is gaining 5 lbs.
	The opposite of spending $20 is earning $20

Try the next two examples on your own and check your answer.

Example 4:	The absolute value of 15 is $\|15\|$ =?
The absolute	distance that 15 is away from 0 is 15, therefore the value of 15 is 15.

Example 5:	What is the opposite of 0?	The opposite of 0 is 0.

Practice

Match the items of the column on the right with those on the left.

_____ 1. 0 a. The temperature outside is a cool 63°

_____ 2. ¯5 b. Marie lost seventeen pounds in 2 months

_____ 3. 2 c. The absolute value of a number is its distance away from _.

_____ 4. ¯17 d. Mark's stock dropped a total of five points in the last quarter

_____ 5. 63 e. The opposite of ¯2

Answers to practice

1. **c.** The absolute value of a number is its distance away from ___.
2. **d.** Mark's stock dropped a total of five points in the last quarter
3. **e.** The opposite of -2
4. **b.** Marie lost seventeen pounds in 2 months
5. **a.** The temperature outside is a cool 63°

Lesson 8b: Addition and Subtraction of Integers

Adding and subtracting integers is similar to adding and subtracting whole numbers except there are a couple of rules to keep in mind. Let's see.

Addition of Integers		
positive + positive	add the integers	answer is positive
negative + negative	add the integers	answer is negative
positive + negative **or** negative + positive	subtract the absolute values of the integers	answer will take the sign of the number with greatest absolute value

For example, 5 + 7 = +12

Adding two positive integers is done in exactly the same way as adding whole numbers. Remember, the positive sign is optional.

For example**,** $^{-}4 + {}^{-}22 = {}^{-}26$

The sum of two negative integers is done in exactly the same way as adding whole numbers with the exception that the negative sign is **not** optional in the answer because $^{-}$**26** is not the same as $^{+}$**26**; they are actually opposites.

Let's take a look at third possibility: when the signs are different.

For example**, $^{-}$17 + 28 = 11**

When the signs are different, as they are in this example, take the <u>difference</u> of the <u>absolute values</u> of the two: the difference between 17 and 28 is 11.

<u>Then</u> take the sign of the integer with the greater absolute value. $|-17| = 17$ and $|28| = 28$. 28 has a greater absolute value and it is positive. Therefore the final answer takes that positive sign.

Another example, **7 + ($^{-}$13) = $^{-}$6**

Since the signs are different, we take the <u>difference</u> of the absolute values of **7** and **$^{-}$13**, which is **6**. The reason why the answer is negative is because **$^{-}$13** has a greater absolute value than **7** and since **13** is <u>negative</u>, the answer is <u>negative</u>.

Subtraction of Integers

When subtracting integers:

(1) change the subtraction to addition

(2) change the <u>sign</u> of the second number

(3) follow the rules of addition.

For example, **12 – 19 = $^{-}$7**

The first step is to <u>change subtraction to addition</u> and then change the sign of the second number: **12 + ($^{-}$19).**

(Keep in mind that 19 was originally $^{+}$.)

Now we use the rules of addition. Since the signs are now different, subtract the two, which is 7. The reason why 7 is negative, is because 19 has a greater absolute value and it is negative. Therefore, the answer is negative.

Example $^{-}5 - (^{-}18) = 13$

Rewrite the problem as addition and change the sign of the second number**: $^{-}5 + (18)$.** Apply the rules of addition: since the signs are different, take the difference, which is 13. The answer is positive because the integer with the greater absolute value is 18 and since it is positive, the answer is positive.

Try the following examples on your own and check your answer.

Example 1: $^{-}24 - (^{-}13)$.

Since this is a subtraction problem, convert it to addition and then change the sign of the second number: $^{-}24 + (+13)$

Since the signs are different, take the difference of 24 and 13, which is 11. The final answer is **$^{-}11$** because 24 has a greater absolute value and it is negative.

Example 2: **$10 - (^{-}16)$.**

Begin by changing subtraction to addition and changing the sign of the second number: $10 + (+16)$

Since the signs are the same and positive, then add and the answer is positive. The final answer is **$^{+}26$**.

Lesson 8c: Multiplication and Division of Integers

Multiplying and dividing integers is similar to multiplying and dividing whole numbers except there are a couple of rules to consider. Unlike addition and subtraction, multiplication and division have exactly the same set of rules. Let's take a look.

Multiplication and Division of Integers	
positive x positive	answer is positive
negative x negative	answer is positive
positive x negative **or** negative x positive	answer is negative

For example, **$5 \times ({}^{-}8) = {}^{-}40$**

Following the rules above, multiply 5 x 8 = 40. A positive x a negative yields a negative answer, therefore the answer is negative.

Another example **${}^{-}16 \div {}^{-}2 = 8$**

Divide 16 by 2 and the answer is 8. The sign is positive because a negative divided by a negative is positive.

Another example **$63 \div 9 = 7$**

This example shows us how dividing two positive integers is exactly like dividing whole numbers. Since the signs are the same, the answer is positive.

Note: The most common mistake made when adding, subtracting, multiplying, and dividing integers is that the rules are often confused among the operations. For example:

1. **when adding two negatives** the sum yields a negative answer
2. **when multiplying two negatives**, the product is positive.

Keep these rules in mind when working through the problems.

Try the next examples on your own and check your answer.

Example 1: **$^-15 \div 5 =$**

Dividing 15 by 5 = 3. The final answer is –3 because a negative divided by a positive is negative.

Example 2: **$[^-3 \times (^-2)] \times 4 =$**

In this example, we need to keep the order of operations in mind. First, complete what is in the brackets and then multiply by 4.

$^-3 \times {}^-2 = 6 \times 4 = 24$

(Multiplying two negatives yields a negative x negative = positive

Example 3: $^-144 \div 12 =$

144 ÷ 12 = 12, but a negative divided by a positive is a negative, therefore, the final answer is **$^-12$**.

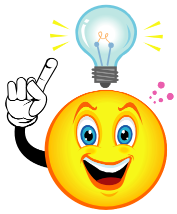
Practice

Match the items of the column on the right with those on the left.

____	1. **0**	**a.** $^{-}7 - (^{-}4)$
____	2. **12**	**b.** $^{-}2 \times (^{-}6)$
____	3. $^{-}$**2**	**c.** $(^{-}8 - 2) \times 3$
____	4. $^{-}$**36**	**d.** $^{-}4 \times [6 - (^{-}3)]$
____	5. $^{-}$**3**	**e.** $8 \div (^{-}2)$
____	6. **8**	**f.** $4 + (^{-}6)$
____	7. $^{-}$**4**	**g.** $0 - (^{-}8)$
____	8. **35**	**h.** $6 + (^{-}6)$
____	9. $^{-}$**30**	**i.** $^{-}7 \times (^{-}5)$
____	10. $^{-}$**5**	**j.** $^{-}7 + 2$

Answers to practice

1. **h.** $6 + (^{-}6)$
2. **b.** $^{-}2 \times (^{-}6)$
3. **f.** $4 + (^{-}6)$
4. **d.** $^{-}4 \times [6 - (^{-}3)]$
5. **a.** $^{-}7 - (^{-}4)$
6. **g.** $0 - (^{-}8)$
7. **e.** $8 \div (^{-}2)$
8. **i.** $^{-}7 \times (^{-}5)$
9. **c.** $(^{-}8 - 2) \times 3$
10. **j.** $^{-}7 + 2$



LESSON 8 THINGS TO REMEMBER

Addition of Integers		
positive + positive	add the integers	answer is positive
negative + negative	add the integers	answer is negative
positive + negative **or** negative + positive	subtract the absolute values of the integers	answer will take the sign of the number with greatest absolute value

For example**, $^{-}$17 + 28 = 11**

When the signs are different, as they are in this example, take the difference of the absolute values of the two: the difference between 17 and 28 is 11. Then take the sign of the integer with the greater absolute value.

$|-17| = 17$ and $|28| = 28$. 28 has a greater absolute value and it is positive. Therefore, the final answer takes that positive sign.

Subtraction of Integers

When subtracting integers:

(4) change the subtraction to addition

(5) change the sign of the second number

(6) follow the rules of addition.

For example, **12 – 19 = $^{-}$7**

The first step is to change subtraction to addition and then change the sign of the second number: **12 + ($^{-}$19).** (Keep in mind that 19 was originally $^{+}$.)

Now we use the rules of addition. Since the signs are now different, subtract the two, which is 7. The reason why 7 is negative, is because 19 has a greater absolute value and it is negative. Therefore, the answer is negative.

Example $^{-}5 - (^{-}18) = 13$

Rewrite the problem as addition and change the sign of the second number**: $^{-}5 + (18)$.** Apply the rules of addition: since the signs are different, take the difference, which is 13. The answer is positive because the integer with the greater absolute value is 18 and since it is positive, the answer is positive.

Multiplication and Division of Integers

Multiplying and dividing integers is similar to multiplying and dividing whole numbers except there are a couple of rules to consider. Unlike addition and subtraction, multiplication and division have exactly the same set of rules.

Multiplication and Division of Integers	
positive x positive	answer is positive
negative x negative	answer is positive
positive x negative **or** negative x positive	answer is negative

For example, **$5 \times (^{-}8) = ^{-}40$**

Following the rules above, multiply 5 x 8 = 40. A positive x a negative yields a negative answer, therefore the answer is negative.

Another example **$^{-}16 \div ^{-}2 = 8$**

Divide 16 by 2 and the answer is 8. The sign is positive because a negative divided by a negative is positive.

Another example **$63 \div 9 = 7$**

This example shows us how dividing two positive integers is exactly like dividing whole numbers. Since the signs are the same, the answer is positive.

Note: The most common mistake made when adding, subtracting, multiplying, and dividing integers is that the rules are often confused among the operations. For example:

- **when adding two negatives** the sum yields a negative answer
- **when multiplying two negatives**, the product is positive.

Keep these rules in mind when working through the problems.

Example 1: $\mathbf{{}^{-}15 \div 5 =}$

Dividing 15 by 5 = 3. The final answer is –3 because a negative divided by a positive is negative.

Example 2: $\mathbf{[{}^{-}3 \times ({}^{-}2)] \times 4 =}$

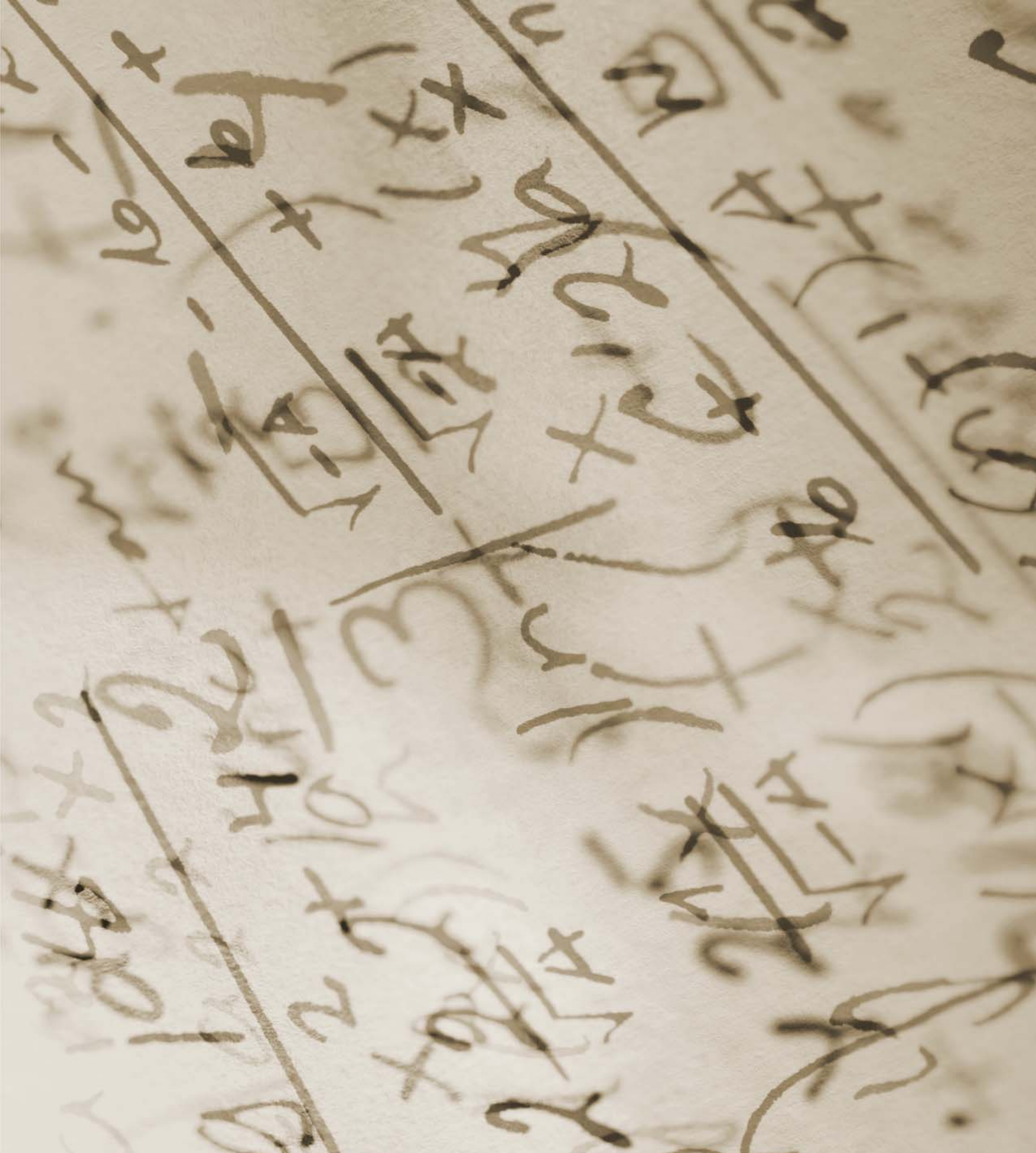
In this example, we need to keep the order of operations in mind. First, complete what is in the brackets and then multiply by 4.

$\mathbf{{}^{-}3 \times {}^{-}2 = 6 \times 4 = 24}$

(Multiplying two negatives yields a negative x negative = positive

Example 3: ${}^{-}144 \div 12 =$

$\mathbf{144 \div 12 = 12}$, but a negative divided by a positive is a negative, therefore, the final answer is $\mathbf{{}^{-}12}$.



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